

# **Estimation of a Microfounded Herding Model On German Survey Expectations**

---

Reiner Franke

# Estimation of a Microfounded Herding Model

## On German Survey Expectations

June 2007

### **Abstract**

The paper considers the dynamic adjustments of an average opinion index that can be derived from a microfounded framework where the individual agents switch between two kinds of sentiment with certain transition probabilities. The index can thus represent a general business climate, i.e., expectations about the future course of the economy. This approach is empirically tested with the survey expectations published by the ZEW and ifo institute. The estimated coefficients make economic sense and are highly significant. In particular, besides effects from fundamental data like the output gap in the recent past, one can identify a strong herding mechanism within both panels, such that metaphorically speaking the agents do not just join the crowd but follow each single motion of it. In addition, the transition probabilities of the ZEW agents are found to be influenced by the ifo climate but not the other way round.

*JEL classification:* D 84, E 32, E 37.

*Keywords:* Herding dynamics, animal spirits, individual transition probabilities, ifo and ZEW climate index, Kalman filter.

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>The dynamic adjustment equation of the climate index</b>	<b>3</b>
2.1	Historical background . . . . .	3
2.2	From microscopic transition probabilities to a macroscopic adjustment equation	4
2.3	Feedbacks in the individual transition probabilities . . . . .	7
<b>3</b>	<b>Estimation of the climate index</b>	<b>12</b>
3.1	The empirical data . . . . .	12
3.2	Estimation of a semi-structural model . . . . .	16
3.3	First estimations of the structural model . . . . .	20
3.4	Dealing with the problem of imprecise estimates . . . . .	24
<b>4</b>	<b>Extensions of the basic specification</b>	<b>28</b>
4.1	Cross effects between the ZEW and ifo panel . . . . .	28
4.2	Estimation with an unobservable variable . . . . .	31
<b>5</b>	<b>Conclusion</b>	<b>35</b>
<b>A</b>	<b>Appendices</b>	<b>37</b>
A.1	A critical benchmark of the majority effect . . . . .	37
A.2	Side calculations . . . . .	38
A.3	Estimation with the extended Kalman filter . . . . .	39

# 1 Introduction

While homogeneous rational expectations are still the ruling paradigm in macroeconomic theory, expectations in the real world are far more diversified and may approximate rational expectations, if at all, only in the aggregate. Being sufficiently self-critical, insecure, and uncertain of the future events, the individual agents are eager to learn about the expectations of others, or about a general climate that is currently prevailing. This is the reason why real-world agents, and financial markets in particular, closely monitor the periodic publications of economic survey indicators.<sup>1</sup>

The evaluation of survey expectations is usually concerned with their ability to predict the future course of the economy. Thus, in the case of Germany and the four surveys available for this country, the ZEW and the ifo expectations indices are held to show the best performance regarding economic growth (see, e.g., Broyer and Savry, 2002), where the ZEW index could be praised to have the highest correlation with industrial production when it is leading five to six months, *vis-à-vis* the ifo expectations with a lead of three or four months (Stadler, 2001; Hüfner and Schröder, 2002a,b). On the other hand, what is lacking in these discussions is a conceptual framework that describes how an opinion index may be formed and how it adjusts over time; with a particular view to possible herding effects of the responding subjects where, for example, optimism feeds optimism. The topic is of more than remote theoretical interest since a pronounced herding mechanism would run counter the abovementioned rational expectations.

One step in the direction of a better understanding of the factors driving the survey expectations is a study by Lahl and Hüfner (2003) on the ZEW Indicator of Economic Sentiment. Using ordinary least squares they find that besides a few lags of this variable itself, each the German manufacturing order data, the German term structure, and the US Consumer Confidence indicator have some additional explanatory power, too, a result that is also confirmed by out-of-sample forecasts.<sup>2</sup> The dynamic adjustments of the ZEW Indicator can thus be described as a combination of *self-reference*, as represented by the

---

<sup>1</sup>It may here also be noted that in contrast to macroeconomic theory which almost exclusively focusses on inflationary expectations, these surveys mainly relate to economic activity as a whole.

<sup>2</sup>Taking up the previous footnote, we find it remarkable that among the other independent variables that the more practically oriented authors explored, they neither included an inflation variable nor a real rate of interest.

significant autoregressive coefficients in the estimation, and of *hetero-reference* (Orléan, 1989). The latter expression means the state of opinion of a social group in its relationship to an external norm, which is here given by a set of central macroeconomic variables in the real, financial and foreign sector.

The investigation by Lahl and Hüfner helps identify basic components in the determination of a survey index. Nevertheless, despite the motivation behind the selection of the explanatory variables, the regression equation is not yet an economic theory. Although it might be tempting to interpret the autoregressive coefficients as reflections of a herding effect, the structure in the regression equation is too poor to warrant such a conclusion. Alternatively, the coefficients may just as well result from some inertia in the adjustment process, or the index itself may move quite in line with other economic variables that are relevant to the survey participants.

This is where the present paper sets in. Reviving and slightly extending a more than 20-years old approach by Weidlich and Haag (1983), it provides a rigorous microfoundation to explain the changes in a climate index such as the ZEW Indicator. For short, this approach may be described as a herding dynamics. It easily captures the self-referential and hetero-referential mechanisms, and it admits of a clearer specification of the, so far, rather vague idea of herding. By virtue of its relatively parsimonious design, the model can be directly estimated by nonlinear least-squares, which we will do for the ZEW as well as for the similarly constructed ifo expectations index. The results may be of interest to understand the dynamics of the two indices and to reveal the features that they share or in which they differ. Our main concern, however, is an empirical validation of our approach to model in a rigorous way such a psychologically contaminated concept as a general business climate, which is akin to the famous ‘animal spirits’.

The remainder of the paper is organized as follows. Section 2 begins with a short overview of the historical background of the model here put forward. It then introduces the agents’ transition probabilities that govern their switches between two opposite attitudes, derives an adjustment equation for the aggregated attitudes, i.e., for the general climate index, and considers several feedback variables with which the transition probabilities may vary. Section 3 tackles the nonlinear least-squares estimations of the basic model thus set up. These results are contrasted with a straightforward linear regression approach, which allows a better understanding of the large standard errors that we first encounter in our

model. The imprecision problems are resolved at the end of Section 3. Section 4 contains two parts. The first one examines whether the transition probabilities of the ZEW agents are also influenced by the ifo climate index, and the other way round. In the second part an additional variable is introduced which is unobservable and to some extent can capture the effects that have so far been omitted. This generalization of the model is estimated by the (extended) Kalman filter. Section 5 concludes.

## 2 The dynamic adjustment equation of the climate index

### 2.1 Historical background

As indicated in the Introduction, the model we propose originates with a stimulating book in the social sciences by Weidlich and Haag (1983). Unfortunately, their approach has not found its way into contemporary macroeconomic theory, although for (heterodox) economists working with feedback-guided macrodynamic systems it would have been an exceptionally fruitful design. In our opinion, two reasons are responsible for this neglect. First, the formulation of the model does not only refer to a probabilistic framework, its analysis also uses concepts from the theory of statistical mechanics like the master equation and the Fokker-Planck equation that are largely unknown to many economists. They are used to study definite time paths of aggregate variables, whereas statistical mechanics is concerned with the evolution of an entire probability distribution or at least, in the mean field approximations, with the time path of expected values. Even the latter concept, however, can be hard to assess, namely, if the stochastic equilibrium of the system is characterized by a bimodal probability density function, in which (otherwise most appealing) case expected values would become meaningless in predicting the likely value of a variable.

A second aspect is insufficient marketing. While the approach was (also) employed in a number of macroeconomic papers, the topics they dealt with were somewhat detached or “exotic” (Kraft et al., 1986; Haag et al., 1987; Weise and Kraft, 1988), or the ordinary reader probably soon drowned in a sea of specification details so that he or she could no longer get hold of the attractive essence of the approach (Weidlich and Braun, 1992). Nevertheless, macroeconomists with a wider area of interest could have also learned from

several related articles by, in particular, Kirman (1993), Lux (1995, 1997, 1998), or Orléan (1995), all of which appeared in highly reputable journals that most of them will have browsed on a regular basis. In sum, the approach by Weidlich and Haag (1983) or similar formulations in the 1990ies offered macroeconomists a good chance to introduce herding dynamics into their models in a very convenient, even standardized way, but this chance was largely missed.<sup>3</sup>

Taking up the original specification by Weidlich and Haag, we content that for our present purpose the whole statistical mechanics apparatus could be dispensed with. Instead, in that language, we can concentrate on a self-contained derivation of the Langevin equation. Accordingly, an ordinary stochastic or deterministic, difference or differential equation will emerge which can subsequently be analyzed, simulated, or estimated like any other adjustment equation of this type.

## 2.2 From microscopic transition probabilities to a macroscopic adjustment equation

Consider a fixed population of  $2N$  agents where at time  $t$  each agent is either optimistic or pessimistic about the future prospects of the economy. Designating an optimistic and pessimistic attitude by  $(+)$  and  $(-)$ , respectively, let  $n_t^+$ ,  $n_t^-$  be the number of optimistic and pessimistic agents at  $t$  ( $n_t^+ + n_t^- = 2N$ ). Next, put  $n_t = (n_t^+ - n_t^-)/2$  and define  $x_t = n_t/N$ . All agents having equal weight in the population, this ratio is the average attitude of agents or, as we will call it, the *climate index*. Clearly,  $-1 \leq x_t \leq 1$ ; optimism and pessimism balance in a state  $x_t = 0$ ; and at  $x_t > 0$  ( $x_t < 0$ ) optimistic (pessimistic) agents form a majority.

---

<sup>3</sup>Taylor and O'Connell (1985), Franke and Asada (1994), and Flaschel et al. (1997, Chapter 12) are three of the few macrodynamic contributions whose central expectational variable is an economy-wide business climate, which is there called a state of confidence. The dynamic adjustments of the latter, however, were formalized in an ad-hoc manner. Without essentially affecting the final results, this part of the models could be easily, and conceptually more satisfactorily, reformulated along the lines propounded in the present paper. Hence, the implicit criticism of not having been sufficiently alert to a fruitful and innovative idea in the past also falls back on the author of this paper, especially since he knew of the article by Weise and Kraft (1988) and Lux (1995) already quite early.

Agents may change their attitude over time. We model this in discrete time and slice time into adjustment periods of length  $\Delta t > 0$ . That is, the agents' attitudes are considered at time  $t$ ,  $t + \Delta t$ ,  $t + 2\Delta t$ , etc. The individual changes will depend on a great variety of idiosyncratic circumstances, which one will not want to specify in all of their details. It rather seems suitable to introduce random elements in this respect, in order to keep the modelling simple and to avoid arbitrary assumptions. Therefore, the basic concept to describe the changes in the climate index are the transition probabilities of the individual agents: at time  $t$ , let  $\pi_t^{-+}$  be the probability *per unit of time* that an agent changes from pessimistic to optimistic, and  $\pi_t^{+-}$  the probability for an opposite change. More exactly,  $\Delta t \pi_t^{-+}$  is the probability that an agent who is pessimistic at  $t$  has become optimistic at the next point in time  $t + \Delta t$ ; and likewise  $\Delta t \pi_t^{+-}$  for an optimistic agent.<sup>4</sup> These probabilities are uniform across the population. They are, however, not fixed but are influenced by the variations of certain macro variables, which will be discussed further below.

Let us beforehand examine how, given  $\pi_t^{+-}$  and  $\pi_t^{-+}$ , the climate index changes from  $t$  to  $t + \Delta t$ .<sup>5</sup> If we consider the 'excess' number of optimistic agents  $n_t$ , it rises by 1 if a pessimistic agent becomes optimistic (when  $n_t^+$  increases and  $n_t^-$  decreases by 1). Symmetrically,  $n_t$  declines by 1 if an optimistic agent turns pessimistic. Denoting by  $k_t^+$  and  $k_t^-$  the number of converts of the first and second type, respectively, we have

$$n_{t+\Delta t} = n_t + k_t^+ - k_t^- \quad (1)$$

As the number of pessimistic agents at time  $t$  can be written as  $n_t^- = N - n_t$ , the number  $k_t^+$  of agents turning optimistic can be viewed as arising from  $N - n_t$  random draws each of which has probability  $\Delta t \pi_t^{-+}$  for the event '+1' (and the complement for the no-change event '0'). The number of these events are then added up. Hence, the random variable  $k_t^+$  has a binomial distribution  $B(N - n_t, \Delta t \pi_t^{-+})$ . Analogously,  $n_t^+ = N + n_t$  being the number of optimistic agents in  $t$ , the random variable  $k_t^-$  is distributed as  $B(N + n_t, \Delta t \pi_t^{+-})$ .<sup>6</sup>

---

<sup>4</sup>Which does not rule out that an agent switches several times within this adjustment period, although this might not appear very plausible for periods of moderate length  $\Delta t$ .

<sup>5</sup>The following argument draws on Alfarano and Lux (2005, Appendix A1 and A2).

<sup>6</sup>A binomial distribution  $B(m, \pi)$  is the probability distribution for the number of successes ( $k$ ) in a sequence of  $m$  independent success/failure experiments, each of which yields success with probability  $\pi$ . The probability of getting exactly  $k$  successes is given by  $\binom{m}{k} \pi^k (1 - \pi)^{m-k}$ , the mean is  $m\pi$ , and the variance  $m\pi(1 - \pi)$ . To be clear, we have presupposed that the individual agents are autonomous, i.e., the realizations of their opinion switching as they are induced by the transition probabilities occur independently of each



The expected values of these variables are  $E(k_t^+) = (N-n_t) \Delta t \pi_t^{-+}$  and  $E(k_t^-) = (N+n_t) \Delta t \pi_t^{+-}$ , their variances amount to  $\text{Var}(k_t^+) = (N-n_t) \Delta t \pi_t^{-+} (1-\Delta t \pi_t^{-+})$  and  $\text{Var}(k_t^-) = (N+n_t) \Delta t \pi_t^{+-} (1-\Delta t \pi_t^{+-})$ . If the expected values are large enough (exceeding 5 or 10), the binomial distributions are (very) well approximated by the Gaussian distributions with the same first and second moments. Taking for granted that the population is large and  $n_t$  not too close to the boundaries  $\pm N$ , we get

$$k_t^+ = E(k_t^+) + \sqrt{\text{Var}(k_t^+)} \xi_t^+, \quad k_t^- = E(k_t^-) + \sqrt{\text{Var}(k_t^-)} \xi_t^- \quad (2)$$

where  $\xi_t^+$  and  $\xi_t^-$  are two independent random draws from the standard normal distribution  $N(0,1)$  (with mean zero and variance equal to one). Furthermore, the difference between two normal distributions yields a normal distribution again. Its mean is the difference between the two single means, its variance the sum of the two single variances. For the random variable  $k_t = k_t^+ - k_t^-$ , we thus have with reference to the climate index  $x_t = n_t/N$ ,  $E(k_t) = \Delta t \cdot [(1-x_t) \pi_t^{-+} - (1+x_t) \pi_t^{+-}] \cdot N$  and  $\text{Var}(k_t) = \Delta t \cdot [(1-x_t) \pi_t^{-+} (1-\Delta t \pi_t^{-+}) + (1+x_t) \pi_t^{+-} (1-\Delta t \pi_t^{+-})] \cdot N$ . It remains to divide (1) and (2) by  $N$  and we obtain,

$$x_{t+\Delta t} = x_t + \Delta t \cdot [(1-x_t) \pi_t^{-+} - (1+x_t) \pi_t^{+-}] + [(\sqrt{\Delta t D_t}/\sqrt{N})] \xi_t \quad (3)$$

$$D_t := (1-x_t) \pi_t^{-+} (1-\Delta t \pi_t^{-+}) + (1+x_t) \pi_t^{+-} (1-\Delta t \pi_t^{+-}), \quad \xi_t \sim N(0,1)$$

Equation (3) abstracts from the many individual and accidental switches in the agents' attitudes and summarizes them in a macroscopic stochastic equation that governs the changes in the climate index. It is the so-called Langevin equation that was announced above (here specified in discrete time). The equation is usually derived by first setting up the entire probability distribution  $P = P[x(t), t; z(\cdot)]$  of  $x$  at time  $t$ , possibly given the time path of a set of exogenous variables  $z$ . To analyze the rate of change of  $P$  the powerful tool of the Fokker-Planck equation (FPE) is employed, which is itself a second-order approximation. Regarding (3), the intimate connection between FPE and the Langevin equation is shown by the fact that the first term in square brackets is the drift coefficient and  $D_t$  corresponds to the fluctuation or diffusion term in FPE.<sup>7</sup>

other (which, depending on the specific social context and its network structure, might not be completely obvious).

<sup>7</sup>See Weidlich and Haag (1983, pp. 22–26) for a succinct presentation of the relationship between FPE and the Langevin equation in continuous time. An example of this treatment in discrete time is Alfarano et al. (2005, pp. 23f, 46f).

On the other hand, if one is not interested in the distribution  $P$  and its evolution over time, the concept of FPE could be circumvented altogether and the story leading to eq. (3) may fully suffice. In fact, the assumptions required for (3) to be valid are not essentially stronger, at least for economists, than those underlying the derivation of FPE.

Three special cases to which the adjustment equation (3) gives rise are easily recognized. First, the noise level decreases with the size of the population and in the limit  $N \rightarrow \infty$ , the herding dynamics becomes a deterministic process (provided  $\pi_t^{-+}$ ,  $\pi_t^{+-}$  do not, directly or indirectly, increase with  $N$ ). Second, the continuous-time limit  $\Delta t \rightarrow 0$  is well-defined, too. If (3) is written as  $x_{t+\Delta t} = x_t + A \Delta t + \tilde{D} \xi \sqrt{\Delta t}$ , then this equation corresponds to the stochastic differential equation  $dx = A dt + \tilde{D} W$ , where  $W$  is a normalized Brownian motion. Lastly, with  $A = A(x, z)$  in this equation, an infinitesimally short adjustment period  $\Delta t$ , and an infinitely large population, the adjustments in (3) ‘degenerate’ to an ordinary differential equation  $\dot{x} = A(x, z)$ .<sup>8</sup> Note that especially the deterministic cases, taken on their own or when incorporated into a more comprehensive framework, could be analyzed like any other difference or differential equation. These remarks show the wide scope of eq. (3) for macrodynamic modelling. All will then hinge on the specification of the transition probabilities, to which we now turn.

## 2.3 Feedbacks in the individual transition probabilities

Generally, the transition probabilities  $\pi_t^{+-}$  and  $\pi_t^{-+}$  between  $t$  and  $t + \Delta t$  will change in response to the variations of a set of several variables that the agents observe. To ease the exposition, let us summarize the variables in a single feedback index  $f_t$ , which can attain positive and negative values in different stages the economy goes through. Positive and negative are related to the probability  $\pi_t^{-+}$  of switching from pessimistic to optimistic, that is, an increase in the feedback index increases  $\pi_t^{-+}$  and decreases the complementary probability  $\pi_t^{+-}$ .

It is an obvious concept, which Weidlich and Haag (1983) have also found very helpful in their formal analysis, to assume that the changes of the transition probabilities depend on the changes of the index  $f_t$  in a linear way. More precisely, the *relative* changes, so

---

<sup>8</sup>More scrupulously, first  $N \rightarrow \infty$  and then  $\Delta t \rightarrow 0$ ; or  $N$  tends faster to infinity than  $\Delta t$  to zero, such that  $\Delta t/N \rightarrow 0$ .

that we have  $d\pi_t^{-+}/\pi_t^{-+} = \alpha df_t$  for some positive constant  $\alpha$ . By suitably scaling  $f$ , this constant can be set equal to one. Symmetry is another natural assumption to make, which gives us  $d\pi_t^{+-}/\pi_t^{+-} = -df_t$ . Introducing  $\nu > 0$  as an ‘integration constant’, the specification of the transition probabilities reads (‘exp’ being the exponential function),

$$\pi_t^{-+} = \pi^{-+}(f_t) = \nu \exp(f_t), \quad \pi_t^{+-} = \pi^{+-}(f_t) = \nu \exp(-f_t) \quad (4)$$

Certainly, (4) ensures positive values of the probabilities. The complementary condition that the feedback index is bounded such that the probabilities are less than unity should be a property of the model into which (4) is incorporated, or the outcome of an empirical estimation.

A special feature of (4) is  $\pi_t^{-+} = \pi_t^{+-} = \nu > 0$  when  $f_t = 0$ . Hence even in the absence of active feedback forces or when the different feedback variables neutralize each other, agents will still change their attitude with a positive probability. These reversals, which can occur in either direction, are to be ascribed to idiosyncratic circumstances; they appear as purely random from a macroscopic point of view and should cancel out in the aggregate. For nonzero values of the feedback  $f_t$ , the coefficient  $\nu$  measures the general responsiveness of the transition probabilities to the arrival of new information. So  $\nu$  can be generally characterized as a *flexibility parameter* (Weidlich and Haag, 1983, p. 41).

While eq. (4) provides a first and useful organizational device, the meaningfulness of the model hinges essentially on the variables that are included in the feedback index. For a basic specification, we concentrate on two variables which, we think, are the most elementary ones to consider. The empirical validity of the model is constituted by estimations in the confines of this setting. Significant results are here also important if we want to sell our approach as a building block ready for implementation in a more encompassing macrodynamic framework, where the user will appreciate a parsimonious specification. Although adding further variables in the feedback index may be informative for specific purposes (two such special issues will be examined later in Section 4), in general one will easily face the problem of arbitrariness: what will be the reason for enriching the feedback index just by this, but not another, variable?

The two variables whose influence on the transition probabilities we investigate are the climate index  $x_t$  itself and a measure of economic activity as a whole. For the latter the concept of the output gap  $y_t$  is employed, i.e., the percentage deviations of actual output

from potential output (the precise definition is given below). We do not only consider the levels of the two variables but, in order to capture possible accelerationist effects, also their rates of change. In this respect, let us now fix the time unit as well as the adjustment period as a month,  $\Delta t = 1$  [month]. Since the agents may guard against the noise that monthly variations can contain, changes over one or several months for  $x_t$  and  $y_t$  are allowed to enter the feedback index, where the corresponding lags  $\tau_x$  and  $\tau_y$  may be distinct. Thus, in a formulation (and dating) that is directly suited for estimation, our model of the business climate reads as follows:

$$x_t = x_{t-1} + \nu [(1-x_{t-1}) \exp(f_{t-1}) - (1+x_{t-1}) \exp(-f_{t-1})] + \varepsilon_{x,t} \quad (5)$$

$$f_{t-1} = \phi_o + \phi_x x_{t-1} + \phi_{\Delta x} \Delta_{\tau_x} x_{t-1} + \phi_y y_{t-1} + \phi_{\Delta y} \Delta_{\tau_y} y_{t-1} \quad (6)$$

$$\Delta_{\tau_u} u_t := (u_t - u_{t-\tau_u}) / \tau_u \quad \text{for } u = x, y \quad (7)$$

Unlike the stochastic perturbations in eq. (3), the random terms  $\varepsilon_{x,t}$  are here assumed to have a constant standard deviation. Actually, the factor  $1/\sqrt{N}$  in (3) will be typically so small that variations in the variance  $D_t$  can there be largely neglected. The present equation (5) rather conceives the  $\varepsilon_{x,t}$  as primarily representing random forces from outside our theoretical framework.

For conceptual reasons another source of randomness should be mentioned here, which has its place still within the specification of the model. These are possible “measurement errors”. In the first instance the term means that the agents, even if their information sets were identical, do not observe the same data as those entering an estimation of (5)–(7). It suffices to touch on the two most important discrepancies: (1) the period- $(t-1)$  macro data may not have been available to the agents at that time or, if so, the data are likely to have been revised by the statistical authorities in the meantime; (2) as will become clear below, the agents had to determine the output gap in different ways from the econometrician.

It is therefore appropriate to include a second random influence  $\varepsilon_{f,t-1}$  in eq. (6), which might also be serially correlated. Since, however, a corresponding amendment would require a more elaborated estimation procedure, we postpone this device until later in Section 4.2. For the time being, any such term  $\varepsilon_{f,t-1}$  is omitted in (6) and we bear in mind that if these effects were relevant, they, too, would be captured by the  $\varepsilon_{x,t}$  perturbations, though possibly not in a fully adequate form.

Accepting the limitation to two explanatory variables, the composition of the feedback index is straightforward. Note first that the output gap  $y$  as well as its rates of change  $\Delta y$  are centered around zero. This allows us to interpret  $\phi_o$  as a *predisposition parameter*, since in a neutral state where  $x_{t-1} = \Delta_\tau x_{t-1} = y_{t-1} = \Delta_\tau y_{t-1} = 0$ , a positive  $\phi_o$  gives rise to a probability  $\pi_t^{-+}$  of switching from pessimistic to optimistic that exceeds  $\nu = \nu \cdot \exp(0)$ , while the reverse probability  $\pi_t^{+-}$  is less than  $\nu$ .<sup>9</sup>

Referring to the expressions that were already mentioned in the Introduction, the feedback of the climate index on itself can be said to represent the notion of self-reference or, in more flowery language, herding, while the impact of the output gap on the climate index is a hetero-referential mechanism. Regarding the herding aspect in the model, we would like to underline that including  $x_{t-1}$  and  $\Delta_\tau x_{t-1}$  in eqs (5), (6) admits an explicit structural interpretation with an immediate psychological plausibility, in contrast to the more technical autoregressive coefficients in the estimation approach to the climate changes by Lahl and Hübner (2003).<sup>10</sup> Nevertheless, the natural presumption that optimism and pessimism are self-reinforcing, which would be reflected by positive coefficients  $\phi_x$  or/and  $\phi_{\Delta x}$ , will still have to be verified by the empirical estimations.

In finer detail, specification (6) distinguishes two variants of herding. A positive coefficient  $\phi_x$  means that the probability of switching from pessimism to optimism is higher, and the reverse probability of switching from optimism to pessimism is lower, the more agents have already converted to an optimistic attitude. The herding effect expressed by  $\phi_x > 0$  may thus be characterized as a *majority effect*. Moreover, on the basis of the arguments given in Appendix 1 the herding effect can be called weak if  $0 < \phi_x < 1$ , and strong if  $\phi_x > 1$  is prevailing.

This notwithstanding, even a strong majority  $x_{t-1}$  may lose its attractiveness if it is already crumbling off. This idea is captured by a positive coefficient  $\phi_{\Delta x}$ , which enables the

---

<sup>9</sup>Weidlich and Haag (1983, p. 41) call their counterpart of  $\phi_o$  a preference parameter. Incidentally, a predisposition of the agents towards optimism does not necessarily imply that optimism dominates pessimism in a stationary state of the adjustment equation (5) (under  $y_{t-1} = \Delta_\tau y_{t-1} = 0$ ). In fact, this will depend on the coefficient  $\phi_x$ : a stationary point  $x^* = 0$  of (5) for  $\phi_o = 0$  is shifted upward by a rising  $\phi_o$  if  $0 < \phi_x < 1$ , and this  $x^*$  shifts downward if  $\phi_x > 1$ ; see Weidlich and Haag (1983, pp. 42–44) and eq. (13) further below.

<sup>10</sup>The authors do not discuss the sign of their autoregressive coefficients, let alone a possible conceptual background.

negative change  $\Delta_{\tau_x} x_{t-1}$  in this situation to have a negative impact on the feedback index  $f_{t-1}$  in (6). Generally,  $\phi_{\Delta x} > 0$  assesses the effect that the probability of switching from pessimism to optimism is high (low) if in the recent past the number of optimistic agents has increased (decreased), from what overall level of optimism or pessimism so ever. In other words, the agents are keeping track of any changes in the current mood of the other agents and (more or less unconsciously) adjust their transition probabilities accordingly. Thus they view the motions of the crowd as an early warning system of future changes; they do not discard these motions as temporary but (in terms of probabilities) respond to them instantaneously. We actually consider this to be herding proper: an individual sheep, to strain the metaphor and neglect the probabilistic setting, does not wait until the great majority of the flock gathers at a greener grass and then joins them, it rather follows the other sheep as soon as they begin moving. A markedly positive coefficient  $\phi_{\Delta x}$  can correspondingly be taken as another manifestation of strong herding, in the sense that it captures the influence of the movements (not position) of the flock. In order to distinguish this effect from high values of the level coefficient  $\phi_x$ , i.e. from the majority effect, and still having the image of sheep in mind, it may also be said that the coefficient  $\phi_{\Delta x} > 0$  measures a *moving-flock effect*.<sup>11</sup>

Turning to the second feedback variable in eq. (6), the basic ideas behind the effects of the output gap  $y_{t-1}$  or its rate of change  $\Delta_{\tau_y} y_{t-1}$  on the business climate are now obvious. Of course, the choice of this economic variable as a possible feedback need not be exclusive (as exemplified by Lahl and Hübner, 2003, who try several other variables in their framework). One should, however, start with a general activity variable, in level or growth rate form or both. First because many other variables of interest may be closely correlated with  $y_{t-1}$  or  $\Delta_{\tau_y} y_{t-1}$ ; and second because this raises the following question: are the agents really able to predict the future course of these variables, or are the predictions rather determined by the recent past of economic activity and its changes? Or is this no contradiction at all?

---

<sup>11</sup>The expression is not meant to carry a connotation of foolishness. At least for certain breeds of sheep this behaviour is rational since it has proved to favour survival and reproduction; and at the head of a moving flock there might be guru. Apart from that, following the herd is just one aspect of sheep-specific (or agent-specific) rationality. Readers of the German sheep crime novel “Glennkill” (“Three Bags Full” in the English translation) by Leonie Swann know that this inclination does not altogether rule out logical and individual conclusions.

In contrast to  $\phi_x$  and  $\phi_{\Delta x}$ , the signs of the two coefficients  $\phi_y$  and  $\phi_{\Delta y}$  related to the output gap are *a priori* ambiguous. On the one hand, high levels of output or above-average growth rates may reassure optimistic agents that their current optimism is justified, and convince pessimistic agents that their fears have become obsolete. In this view,  $\phi_y \geq 0$ ,  $\phi_{\Delta y} \geq 0$ . On the other hand, such a situation might also be interpreted as carrying the seeds of a future slowdown or even downturn. For example, the central bank could be expected to raise interest rates, unless it has already done so. These fears of diminishing business prospects would be reflected by negative coefficients  $\phi_y$  or/and  $\phi_{\Delta y}$ . It is thus an open and interesting problem whether the two coefficients would come out significant in an empirical estimation, and if so, how they are signed.

### 3 Estimation of the climate index

#### 3.1 The empirical data

As mentioned in the Introduction there are two leading sentiment indicators for the German economy. These surveys, which are regularly published on a monthly basis, are carried out by the ifo institute and the ZEW institute.<sup>12</sup> While the institutes ask a series of questions and construct several indices, we focus on the respondents' expectations of the general business situation six months ahead.<sup>13</sup> This horizon is the same for both institutes, and both of them categorize the answers into the options 'better', 'worse' or 'just about the same', of which they report the difference between the percentages of 'better' and 'worse' answers. The index is thus in relatively good concordance with our specification of the climate variable  $x$  in Section 2.2.

The main difference between the two surveys are the number of participants and the economic sectors from which they are recruited. The ifo institute asks more than 7,000 business leaders and senior managers from all sectors except the financial sector (the answers are weighted according to the importance of the single industries). In contrast, the

---

<sup>12</sup>See <http://www.ifo.de> and <http://www.zew.de>. In German, the 'ifo' institute usually presents itself in lower-case characters.

<sup>13</sup>Especially regarding the ifo institute, this index should not be confused with the climate or sentiment index that is occasionally referred to in the mass media, as the latter is concerned with an evaluation of the current business situation.

ZEW survey echoes the opinion of the German financial sector, i.e., the participants are financial analysts and institutional investors from banks (comprising 77% of the sample), insurance companies, and large industrial corporations. The number of subjects contacted is about 350.

The ZEW survey has started later than the ifo survey, in December 1991. The data we are using end with 2006:6. To get a first impression of the two expectation indices, the series are plotted together in the top panel of Figure 1, where they are rescaled in order to fit into the interval between  $-1$  and  $+1$ . For a better comparison of the results and to be sufficiently bounded away from these end-points (cf. the derivation of eq.(2)), the factor by which the original series are multiplied is tuned such that the two indices are contained within an interval  $\pm 0.80$ . The ifo index attains a corresponding minimum in 1992:11 (plotted at  $t = 1992.83$ ), the ZEW index a maximum in 2000:1.<sup>14</sup>

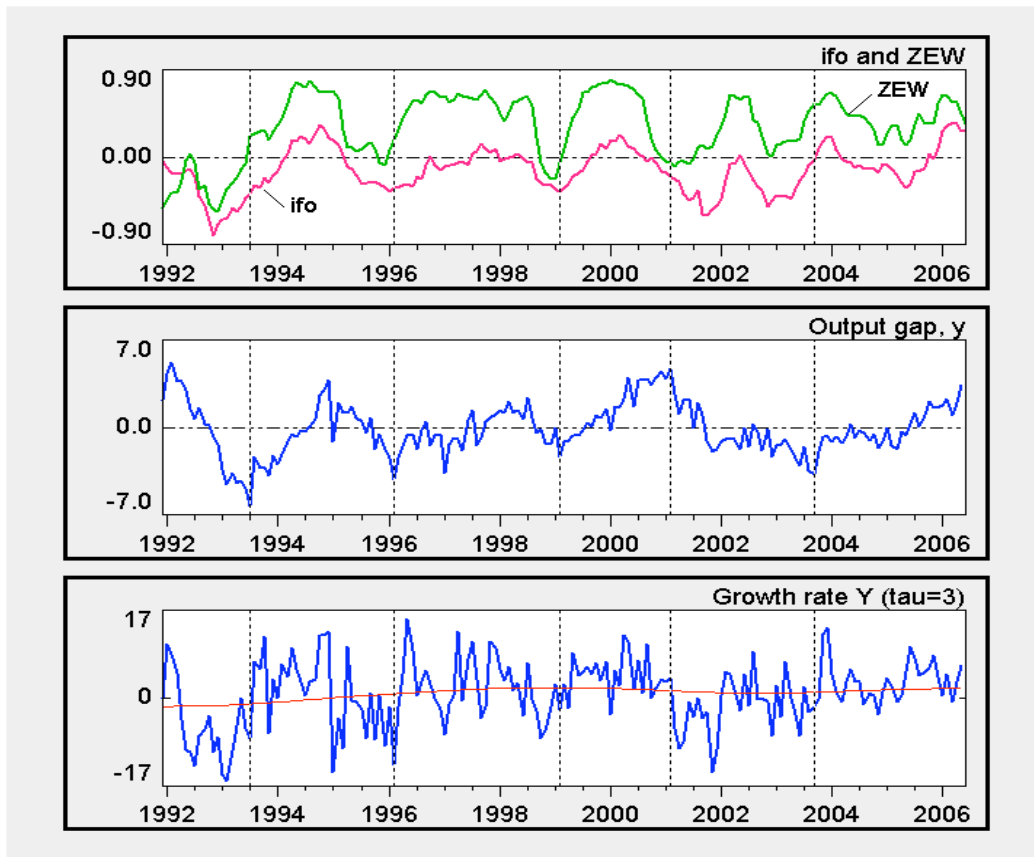
At a first glance, the two indices exhibit a similar and clearly cyclical pattern, possibly with a tendency for the ZEW index to be slightly leading in the turning points. An obvious difference are the levels of the two series, where from the second half of 1992 onward the ifo index runs persistently below the ZEW index; the mean values are  $-0.135$  and  $0.329$ , respectively. From this descriptive point of view the ifo index can be characterized as basically pessimistic, the ZEW index as optimistic. For our structural model it may be expected that this difference finds expression in negative and positive predisposition parameters  $\phi_o$ .

The lower two panels of Figure 1 show the behaviour of the variable that is to represent the model's component of hetero-reference. As indicated in Section 2, we choose output for this purpose. Specifically, we work with industrial production, since it is the only output category available as monthly data and was also used in other work (Hüfner and Schröder, 2002a,b; see, in addition, the discussion in Broyer and Savry, 2002). For an export-oriented country like Germany with its weak and passive domestic demand, the manufacturing sector can, however, still be regarded as providing the basic impulses for the rest of the economy, especially for the large subsector of the business-related services (Franke and Kalmbach, 2005). Manufacturing output therefore contains more information about economic activity as a whole than its share of about one-third in the national product might suggest.

---

<sup>14</sup>Here and in the following it will be understood that when we refer to the 'ifo index' or 'ZEW index', the rescaled magnitudes are meant.





**Figure 1:** Time series of the data.

*Note:* The numbers in the second and third panel are percentages. The output variable  $Y$  there is monthly industrial production. Detrending of log output uses the Hodrick-Prescott filter with smoothing parameter  $\lambda = 120,000$ ; the growth rates are annualized 3-month changes. The thin line in the bottom panel is the trend growth rate implied by the filter.

The middle panel displays the output gap  $y_t$  for this variable. It is defined as the percentage deviations of output from trend. The trend line is obtained by applying the flexible Hodrick-Prescott filter to log output, with a smoothing parameter  $\lambda = 120,000$ . The reason for employing a roughly 8 times higher value than the conventional value of 14,400 is the variability in the implied trend growth rate (i.e., the slope of trend log output). As can be seen from the thin line in the bottom panel, trend growth still exhibits sizeable movements (though this is optically downplayed by the large fluctuations of the main series in that panel): rising from  $-1.6\%$  in 1992 to  $1.9\%$  in 1998, falling to  $0.9\%$  in 2002 and increasing again to  $1.8\%$  at the end of the sample. While one might wonder whether these variations still justify the notion of ‘trend growth’, the variability would be even more

severe for the usual  $\lambda = 14,400$ . Nevertheless, since the deviations of the two- or three-month output growth rates from the trend rates are fairly large, and these deviations will constitute the variable  $\Delta_{\tau_y} y_t := (y_t - y_{t-\tau_y}) / \tau_y$  from eq. (7) in our estimations below, we can do without a stronger smoothing of the trend and maintain  $\lambda = 120,000$ . The bold series in the bottom panel is in fact the annualized three-month growth rate of output.

A visual inspection of the comovements of the output gap with the two expectation indices shows a (near-) coincidence of the troughs of  $y_t$  and the ifo index in 1996:2 and 1999:2, slightly led by the ZEW index (see the vertical dashed lines in Figure 1). At the two other troughs of  $y_t$  in 1993:7 and 2003:9, however, both indices are already rising for more than six months (at least). Conversely, at the output peak in 2001:2 the indices are on the downturn, the ZEW index being even just about to reach its next trough. Hence there is no obvious pattern of synchronized movements, contemporaneously or lagged, of the output gap and the expectation indices.<sup>15</sup>

In discussions of the indices it is rather more usual to compare them to the growth rates of output. Regarding the 12-month growth rates ( $g_t$ ) of industrial production over the sample period 1992:1–2002:3, Hüfner and Schröder (2002a) report a significant lead of  $\theta = 2$  months for the ifo index and  $\theta = 5$  months for the ZEW index, which they infer from the maximal cross-correlation coefficients  $\text{Corr}(g_t, x_{t-\theta}) = 0.88$  and  $0.81$ , respectively.<sup>16</sup> Adding the last four years to these observations and computing the cross-correlations  $\text{Corr}(\Delta_{12} y_t, x_{t-\theta})$  over our period 1991:12–2006:6, the first part of Table 1 largely confirms these results, though the coherence is weaker and the leads are one month shorter (the bold figures in the table indicate maximal coefficients). It is in this sense that the institutes praise the predictive power of their indices.

It should be emphasized at this point that although lagged values of  $x_t$  are correlated with current growth rates of output or the output gap, for that matter, this does by no means rule out that lagged output growth rates may also determine current expectations. One reason is that in our equation (5)  $x_t$  is determined jointly by several variables, the

---

<sup>15</sup>Statistically there are nevertheless cross-correlations  $\text{Corr}(y_t, x_{t-\theta}) = 0.47$  at  $\theta = 6$  for the ZEW index and  $\text{Corr}(y_t, x_{t-\theta}) = 0.60$  at  $\theta = 5$  for the ifo index, though this has not been sold as forecasting evidence so far. In fact, given the relatively smooth character of the series, the coefficients should be somewhat higher than that.

<sup>16</sup>As concerns the lags there is a sign error in their Table 1 on p. 6.

	Lead $\theta$						
	4	3	2	1	0	-1	-2
<hr/>							
$\Delta_{12} y_t$							
ifo:	0.65	0.70	0.71	<b>0.71</b>	0.68	0.65	0.61
ZEW:	<b>0.70</b>	0.69	0.56	0.50	0.43	0.37	0.31
<hr/>							
$\Delta_3 y_t$							
ifo:	0.17	0.26	0.34	0.41	0.45	<b>0.49</b>	0.48
ZEW:	0.34	0.38	0.37	0.39	0.41	<b>0.42</b>	0.41
<hr/>							

**Table 1:** Cross-correlations of index  $x_{t-\theta}$  with growth rates of monthly industrial production.

other reason that there, more precisely, lagged output growth does not act on the levels  $x_t$  but on the first differences  $x_t - x_{t-1}$  of the expectation index.

In addition, the second part of Table 1 demonstrates that the forecasting capabilities of the indices vanish if we refer to quarterly changes of output. Here, if anything, it is the recent changes  $\Delta_3 y_{t-1}$  that determine current expectations  $x_t$ , rather than  $x_{t-\theta}$  predicting  $\Delta_3 y_t$ . In any case, whether lagged levels or growth rates of output can explain current expectations, though jointly with a self-reference mechanism of the latter, must be an issue of specific estimations.

### 3.2 Estimation of a semi-structural model

Before estimating the nonlinear modelling equations (5)–(7), it is useful to study a linear regression approach that includes the same set of explanatory variables. In its general form, we have the following ordinary least squares (OLS) estimation problem,<sup>17</sup>

$$x_t = \beta_o + \sum_{\theta \geq 1} \beta_\theta x_{t-\theta} + \sum_{\theta \geq 1} \gamma_\theta y_{t-\theta} + \varepsilon_t \quad (8)$$

---

<sup>17</sup>Regarding the scaling of the output gap in the estimation equations, a percentage deviation of, for example, 1.5% enters as  $y = 0.015$ .

The first month of the sample period underlying (8), which is the same for both the ZEW and ifo expectations, is  $t = 1992:3$ , the last month is  $t = 2006:6$ . This amounts to a total of 172 observations.

Equation (8) has a minimal interpretation, in that it excludes any hypothesis of rational expectations (no forward-looking variable on the right-hand side) and states that the climate is determined by distributed lags just of itself and the output gap, and no other variables. Beyond this, no further economic structure is made explicit. Linearity is then the most convenient, if not only meaningful, assumption to make. The equation can be thought of as being compatible with one or several (possibly linearized) model formulations of business climate adjustments, which may already, or may not yet, exist. From this point of view, the formulation in (8) is best characterized as a semi-structural model. On the other hand, (8) can be given a specific interpretation if it is regarded as a linearization of our adjustment equation (5) (eq. (10) below will provide the details).

The regressions in (8) are treated in several stages. The first stage is given by the benchmark of a random walk process, according to which all coefficients are zero except  $\beta_1$ , which is fixed at unity. Beginning with the ZEW index, the resulting root mean square error (RMSE) is reported in the first row of Table 2.

The second statistic in the table is two times the value  $L$  of the log-likelihood of the residuals  $\varepsilon_t$  (see, e.g., Davidson and MacKinnon, 2004, p.403). Applying the likelihood ratio (LR) test (ibid., pp.420f), this value will help us decide whether an estimation B with  $r_B$  independent variables is significantly better than a previous estimation A with  $r_A < r_B$  variables. The criterion is the difference (in obvious notation)  $LR = 2 [L(B) - L(A)]$ , which for a large enough sample is approximately distributed as chi-square with  $r = r_B - r_A$  degrees of freedom. As can be read from any formulary, estimation B is significantly better for  $r = 1$  and  $r = 2$  at a 95% level if  $LR > \chi_{0.95}^2(1) = 3.84$ , or if  $LR > \chi_{0.95}^2(2) = 5.99$ , respectively.<sup>18</sup>

With this yardstick at hand, it can be checked step by step if additional regressors yield a significant improvement. Compared to the random walk, there is no doubt about

---

<sup>18</sup>While the comparison of two log-likelihoods provides us with a statistical decision criterion, the root mean square error gives a more direct assessment of the order of magnitude of the one-step prediction errors. In Table 2 and the other tables to follow, RMSE will therefore be reported as a complementary measure of the goodness-of-fit, though we will no longer comment on it.

	$\beta_o$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\gamma_y$	$\gamma_{\Delta y}$	$\tau_y$	2L	RMSE
<u>ZEW expectations</u>										
1	--	1.00	--	--	--	--	--	--	307.1	0.0994
2	0.027 (0.009)	1.45 (0.06)	-0.53 (0.06)	-- .	--	--	--	--	372.9	0.0821
3	0.028 (0.009)	1.44 (0.08)	-0.54 (0.13)	0.07 (0.12)	-0.06 (0.07)	-- .	--	--	373.7	0.0819
4	0.024 (0.009)	1.43 (0.06)	-0.50 (0.07)	-- --	-- --	-0.55 (0.32)	-- .	--	376.0	0.0814
5	0.034 (0.009)	1.37 (0.07)	-0.48 (0.06)	-- --	-- --	-0.92 (0.33)	4.41 (1.43)	3	385.3	0.0792
6	<b>0.036</b> (0.009)	<b>1.37</b> (0.07)	<b>-0.48</b> (0.06)	-- --	-- --	<b>-1.04</b> (0.34)	<b>5.90</b> (1.78)	<b>4</b>	<b>386.9</b>	<b>0.0788</b>
<u>ifo expectations</u>										
1	--	1.00	--	--	--	--	--	--	407.3	0.0743
2	-0.013 (0.006)	1.11 (0.03)	-- --	-- --	-0.21 (0.03)	--	--	--	444.8	0.0666
3	-0.012 (0.007)	1.11 (0.03)	-- --	-- --	-0.20 (0.04)	-0.16 (0.31)	--	--	445.1	0.0665
4	<b>-0.020</b> (0.007)	<b>1.04</b> (0.04)	-- --	-- --	<b>-0.19</b> (0.04)	<b>-0.63</b> (0.32)	<b>7.68</b> (1.90)	<b>6</b>	<b>460.7</b>	<b>0.0636</b>

**Table 2:** OLS estimations of regression (9).

*Note:* Numbers in parentheses are the standard errors, RMSE is the root mean square error of the predictions from the right-hand side of (9). 2L is 2 times the value of the log-likelihood function. Thus, an estimation in a row with one (two) additional independent variable(s) is significantly better at a 95% level than that in another row if the difference exceeds 3.84 (or 5.99, respectively).

including a constant and the first two lags of the ZEW index; see the second row of Table 2. From this estimation on, (first-order) autocorrelation in the residuals is negligible, so no Durbin-Watson or a similar statistic is reported.

Actually, we start out from a regression of  $x_t$  on a constant and  $x_{t-1}$ . The two coefficients  $\beta_o$  and  $\beta_1$  are both highly significant. Then we consecutively add one further lag  $x_{t-\theta}$  for  $\theta = 2, 3, \dots$ , and choose the lag with the strongest improvement in the likelihood.

Complementarily, we add a large number of lags simultaneously and choose the lag with the most significant coefficient. The result is in both cases the same, namely, lag  $\theta = 2$ .

On this basis, with a constant and  $x_{t-1}$ ,  $x_{t-2}$  as regressors, we proceed in a likewise manner to see whether another lag of  $x_t$  can improve upon this estimation. Remarkably, this is not possible. The third row in Table 2 exemplifies this for adding lags  $\theta = 3, 4$ , and the results are quite similar for other combinations of lags (up to  $\theta = 13$ ).

We note that the significance of only two lags of  $x_t$  in eq. (8) is a good support for the specification of auto-reference in the feedback index of eq. (6), which is equally limited to two lags of the climate. In fact, a sum  $\beta_1 x_{t-1} + \beta_{1+\tau} x_{t-1-\tau}$  in (8) can be equivalently written as  $(\beta_1 + \beta_{1+\tau}) x_{t-1} - \tau \beta_{1+\tau} \Delta_\tau x_{t-1}$  if a rate of change is invoked ( $\Delta_\tau x_{t-1}$  as defined in (7)).

As far as the self-reference in (8) is concerned, we therefore settle down on a constant and the two lags  $x_{t-1}$ ,  $x_{t-2}$ . Then, similar as above, we check whether, and with which lags, the output gap can enhance the regression. The fourth row in Table 2 shows that including the output gap  $y_{t-1}$  misses the LR criterion ( $376.0 - 372.9 = 3.1 < \chi_{0.95}^2(1) = 3.84$ ;  $\gamma_y$  in the head of the table corresponds to  $\gamma_1$  in (8)). However, if we here do not stop short but still add another, suitable lag, we find that it comes out significant and that now  $y_{t-1}$  is significant, too (it is discussed in a moment how this result shows up in Table 2). The greatest gain in this respect is achieved with lag  $\theta = 5$ . Moreover, all additional lags of the output gap (including its contemporaneous value with  $x_t$ ) lead to no further significant improvement.

Hence, also the specification of hetero-reference in our model's feedback equation (6), with one lagged level and one rate of change (as of  $t-1$ ) of the output gap, is well supported by the investigation of regression (8). For a slightly more convenient presentation of the two-lag significance results and other estimations in Table 2, we formulate eq. (8) as

$$x_t = \beta_o + \sum_{\theta=1}^4 \beta_\theta x_{t-\theta} + \gamma_y y_{t-1} + \gamma_{\Delta y} \Delta_{\tau_y} y_{t-1} + \varepsilon_t \quad (9)$$

The sixth row of Table 2 presents the optimal fit that, as just mentioned, is achieved with  $\tau_y = 4$ . Its optimality is pointed out by the bold face figures. The outcome for the more common quarterly lag,  $\tau_y = 3$ , in the rate of change of the output gap is given before in the fifth row, which is only slightly inferior. The estimated coefficients are not much different,

either. At least part of the lower coefficient on  $\Delta_{\tau_y} y_{t-1}$  can be explained by the fact that these changes are larger for the quarterly lag than for  $\tau_y = 4$ .

The second part of Table 2 documents the central results from carrying out the same battery of estimations for the ifo index. Generally, this series admits a better fit than the ZEW index, as can be seen from its lower RMSEs and higher values of the log-likelihood. It bears emphasizing that for both self- and hetero-reference, again only two lags prove to be significant. With this additional support, the specification of the feedback equation (6) can be claimed to be built on firm ground. The only difference between the ifo and ZEW expectations are the lags involved:  $\theta=4$  versus  $\theta=2$  in the self-reference mechanism (which corresponds to  $\tau_x=3$  versus  $\tau_x=1$ ), and  $\tau_y=6$  versus  $\tau_y=4$  regarding the influence of the output rates of change.

### 3.3 First estimations of the structural model

After the investigation of the semi-structural adjustment equation of the climate index, we can now return to the original model (5)–(7). Plugging (6), (7) into (5), we have a regression that can be directly estimated by nonlinear least-squares (NLS).<sup>19</sup> Concentrating on the optimal lags of the explanatory variables from Table 2, the outcome is summarized in Table 3.<sup>20</sup>

Let us again begin with the ZEW expectations. The first row corresponds to the second row in Table 2 and presents an estimation where the model is confined to its herding component. Besides the expected positive predisposition parameter,  $\phi_o > 0$ , both herding coefficients  $\phi_x$  and  $\phi_{\Delta x}$  have the correct positive sign. However, the significantly larger likelihood statistic in the second row ( $2L = 385.2$ ) makes it clear that, as before, these

---

<sup>19</sup>In an alternative attempt, Lux (2007) goes back to the micro level, sets up the abovementioned Fokker-Planck equation in (essentially) continuous time and computes from there the conditional transitional probability densities between two months, which can then be used for a maximum likelihood estimation. This approach is potentially superior since it seeks to exploit more information, though this goes at the price of a high computational effort and also its conceptual basis might slightly differ in detail. As yet, however, the relationship between the straightforward NLS approach of (5)–(7) and this more ambitious approach are not sufficiently well explored.

<sup>20</sup>We have checked that the optimality of these lags is indeed maintained in the estimation of the structural model.

effects should be complemented by the feedbacks from the output gap. That is, the herding mechanism has to be augmented by a mechanism of hetero-reference.

If the resulting likelihood is compared to the corresponding  $2L = 386.9$  in the sixth row of Table 2, it seems that the linear regression performs somewhat better, though it is not significantly superior (the issue of the number of coefficients will be discussed in a moment). It can thus be stated that, at least as far as the goodness-of-fit is concerned, the structural model (5)–(7) is supported by the data. In addition, it will later be shown that the fit can be substantially improved if the role of the random perturbations is slightly modified.

It may also be observed that in contrast to the growth rate coefficient  $\phi_{\Delta y}$ , the coefficient  $\phi_y$  on the level of the output gap is negative. According to the interpretation at the end of Section 2, the negative sign expresses certain doubts of the agents that a prosperous phase of the economy will be sustained; or their hopes in a slump that the economy will be able to recover.

Even if these results make good economic sense, the standard errors in parentheses in the second row of Table 3 indicate that a great deficiency still remains, namely, the imprecision of the estimates. If we follow the usual econometric standards then none of the six coefficients is significantly different from zero. This is especially annoying for the flexibility parameter  $\nu$ , since  $\nu = 0$  would be completely meaningless.

The imprecision of the coefficients may not come unexpectedly if the structural equations (5)–(7) are compared to the linear regression equation (9). The two approaches differ on two counts: the structural model is nonlinear; and it contains one parameter “too many”, in the sense that (9) and (5)–(7) share the feature that the influence of each of the (common) explanatory variables is weighted by a coefficient, whereas  $\nu$  in (5) plays an extra role as it is additionally supposed to measure the strength of the composite effect. This means that if the term in square brackets in (5) were linear in the feedback index  $f_{t-1}$ , or nearly linear, it would not be possible to identify  $\nu$  separately from the other  $\phi$ -coefficients. Under these circumstances we would actually have the following approximate relationship between these coefficients and the parameters in (9) (neglecting the intercepts  $\phi_o$  and  $\beta_o$ ):

$$\begin{aligned} 1 + \nu(c_1\phi_x - c_2) &\approx \beta_1 + \beta_{1+\tau_x} & c_1\nu\phi_{\Delta x} &\approx -\tau_x\beta_{1+\tau_x} \\ c_1\nu\phi_y &\approx \gamma_y & c_1\nu\phi_{\Delta y} &\approx \gamma_{\Delta y} \end{aligned} \tag{10}$$



	$\nu$	$\phi_o$	$\phi_x$	$\phi_{\Delta x}$	$\phi_y$	$\phi_{\Delta y}$	2 L	RMSE
<u>ZEW expectations</u>								
1	0.110	0.117	0.78	2.46	—	—	372.0	0.0823
2	0.093 (0.073)	0.195 (0.153)	0.52 (0.48)	2.54 (1.75)	−5.42 (4.49)	32.8 (25.8)	385.2	0.0792
3	0.082 (0.054) [—]	0.205	0.41 (0.45) [0.14]	2.83 (1.59) [0.35]	−6.23 (4.27) [2.05]	37.3 (24.6) [11.0]	384.8	0.0793
4	0.092 (0.063)	0.107	0.71	2.69	−2.78	—	373.8	0.0819
5	0.085 (0.058)	0.174	0.51	3.11	—	20.8	375.9	0.0814
6	0.037 (0.034)	0.396	−0.17	—	−22.07	81.7	338.2	0.0908
7	<b>0.053</b> (0.011)	<b>0.342</b>	—	<b>3.86</b> (0.74)	<b>−9.05</b> (3.05)	<b>57.7</b> (14.0)	<b>383.7</b>	<b>0.0795</b>
<u>ifo expectations</u>								
1	0.196 (0.127)	−0.044	0.68 <b>(0.26)</b>	1.49 (1.01)	−1.68 (1.36)	20.5 (13.7)	462.2	0.0633
2	0.072 (0.016)	−0.135	—	4.02 (0.89)	−4.15 (2.64)	53.7 (13.2)	460.8	0.0636
3	<b>0.081</b> (0.015)	<b>−0.135</b>	—	<b>4.27</b> (0.82)	—	<b>39.9</b> (8.9)	<b>457.1</b>	<b>0.0643</b>

**Table 3:** NLS estimations of the structural model (5)–(7).

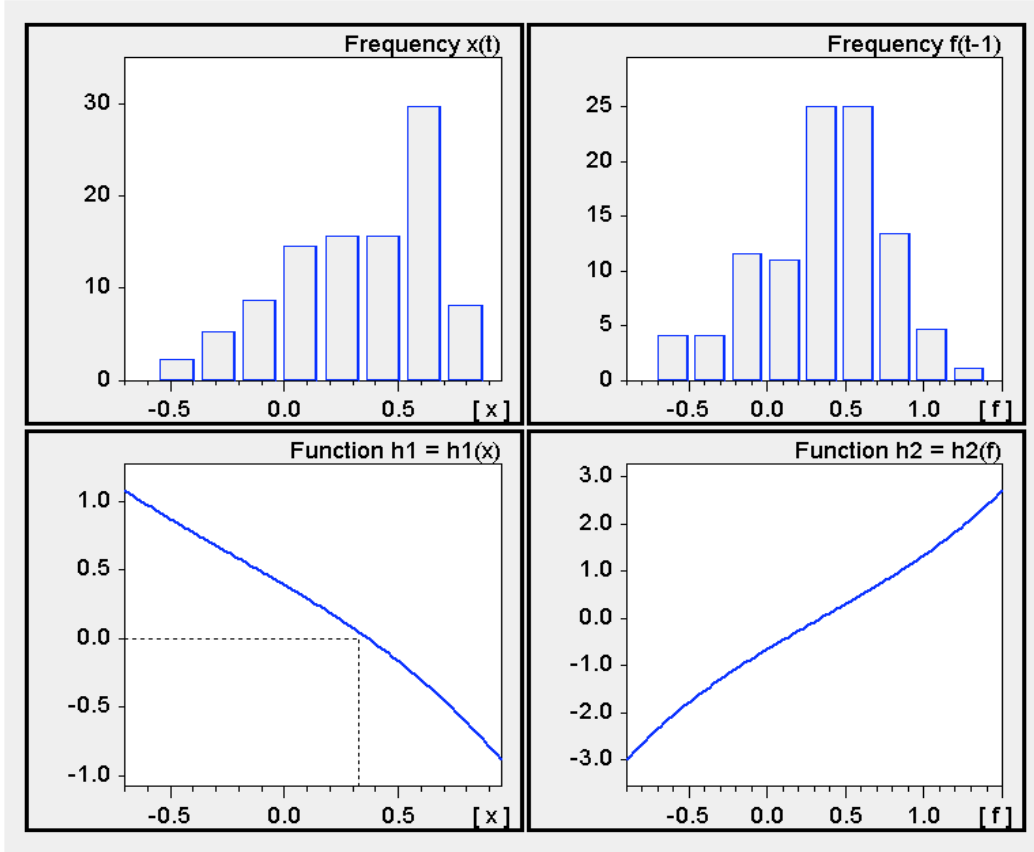
*Note:* Numbers in parentheses are the standard errors of the estimates. Except for the first two rows in the upper part of the table, the predisposition parameter  $\phi_o$  is determined by condition (13),  $\phi_o = \phi_o^e(\phi_x, \bar{x})$ . The lags underlying are  $\tau_x = 1$ ,  $\tau_y = 4$  for the ZEW index, and  $\tau_x = 3$ ,  $\tau_y = 6$  for the ifo index.

where  $c_1$  and  $c_2$  are two constants close to 2 that depend only on  $\bar{x}$ , the sample mean of the climate index ( $c_1 = 2\sqrt{1-\bar{x}}\sqrt{1+\bar{x}}$ ,  $c_2 = 2/[\sqrt{1-\bar{x}}\sqrt{1+\bar{x}}]$ ; see Appendix 2 for the details). Obviously  $\nu$  could here be treated as a free parameter that essentially scales the other coefficients in the structural model; given the estimates of the  $\beta$ - and  $\gamma$ -coefficients in the linear version and a chosen level of  $\nu$ , all of the  $\phi$ -coefficients are unambiguously determined.

This observation raises the question for the degree of nonlinearity in the structural model. To get an impression of the curvature, two versions of the main functional expression in (5) are considered, which differ in the *ceteris paribus* specifications to set up a one-dimensional mapping. First, we employ the estimates  $\phi_o$  and  $\phi_x$  from the second row of the ZEW index in Table 3, freeze  $y$  and  $\Delta_{\tau_y}y$  at zero, and plot the graph of the function

$$h_1 = h_1(x) = (1-x) \exp(\phi_o + \phi_x x) - (1+x) \exp(-\phi_o - \phi_x x) \quad (11)$$

This is done in the lower-left panel of Figure 2. The panel above it draws the frequency distribution of the empirical ZEW values of  $x_t$ . It is seen in this way that at least 90 percent of the observations of  $x_t$  fall into the quasi-linear segment of function  $h_1$ . Things are very similar if the same check is made or the ifo index.



**Figure 2:** Plots of the functions  $h_1 = h_1(x)$  and  $h_2 = h_2(f)$ .

*Note:* The two top panels draw the frequency distributions of the empirical values of the ZEW index  $x_t$  and its feedback index  $f_{t-1}$  (derived from the estimated coefficients in row 2 of Table 3). The dotted line in the lower-left panel marks the sample mean  $\bar{x} = 0.329$  of the ZEW index.

In a second illustration, the index of the ZEW is fixed at the sample mean  $\bar{x} = 0.329$ . We let the entire feedback index  $f$  vary and, in the lower-right panel in the figure, plot the function defined by

$$h_2 = h_2(f) = (1 - \bar{x}) \exp(f) - (1 + \bar{x}) \exp(-f) \quad (12)$$

This function, too, is almost linear over the range that matters, as indicated by the top-right panel that draws the frequency distribution of the empirical values of the feedback index  $f_{t-1}$  in eq. (6) (likewise computed on the basis of the coefficient estimates in the second row of Table 3). So it must be concluded from Figure 2 that the problem raised by the relationships in (10) is indeed a relevant problem.<sup>21</sup>

### 3.4 Dealing with the problem of imprecise estimates

Before proceeding with the discussion, let us save one parameter. This is done for conceptual reasons and to simplify the computations in the estimations, but it will not solve our problem. If we have a look at the lower-left panel of Figure 2, it is seen that at the empirical mean value of the ZEW index,  $\bar{x} = 0.329$ , the function  $h_1$  nearly vanishes, which would make  $\bar{x}$  a point of rest in the adjustment equation (5). This observation motivates us to postulate directly that in the presence of  $y_{t-1} = \Delta_{\tau_y} y_{t-1} = \Delta_{\tau_x} x_{t-1} = 0$ , the sample mean of the climate index also constitutes an equilibrium of (5). Involved are here the two coefficients  $\phi_o$  and  $\phi_x$ , which, given  $\bar{x}$ , are linked by the condition  $h_1(\bar{x}) = 0$  in (11). Rearranging the terms in the equation as  $(1 - \bar{x})/(1 + \bar{x}) = \exp(-\bar{f})/\exp(\bar{f}) = \exp(-2\bar{f})$  (where  $\bar{f} = \phi_o + \phi_x \bar{x}$ ) and taking logs, the equilibrium condition can be explicitly solved for  $\phi_o$ . Denoting this value by a superscript ‘e’, we get

$$\phi_o^e = \phi_o^e(\phi_x, \bar{x}) = - \left[ \frac{1}{2} \ln \frac{1 - \bar{x}}{1 + \bar{x}} + \phi_x \bar{x} \right] \quad (13)$$

Technically speaking,  $\phi_o^e$  is the intercept in the feedback index that establishes  $\bar{x}$  as an equilibrium point of the climate dynamics (5). Conceptually, (13) determines the predisposition parameter of the agents from the average climate and the estimate of  $\phi_x$ . Note that the log expression is approximately  $(1 - \bar{x} - 1) - (1 + \bar{x} - 1) = -2\bar{x}$ . Hence  $\phi_o^e \approx (1 - \phi_x) \bar{x}$

---

<sup>21</sup>As a side result, Figure 2 ensures us that the original transition probabilities in (4),  $\pi_t^{-/+} = \nu \exp(\pm f_t)$ , are well specified; given the order of magnitude of the estimated  $\nu$ , the condition that they are less than unity is always safely satisfied.

and, provided that  $\phi_x$  is less than unity, a positive (negative) sample mean of the climate index is, in the structural model, indeed indicative of a predisposition of the agents toward optimism (pessimism).

For a nonlinear regression it is no additional problem to replace the coefficient  $\phi_o$  in (5) and (6) with the value of  $\phi_o^e$  in (13) that is linked to  $\phi_x$ . The estimation result for the ZEW index is reported in the third row of Table 3. Apparently, as a comparison with the likelihood in the second row shows, the requirement that the conceptual equilibrium value of  $x$  coincides with the sample mean is not a very strong constraint on the data. However, as noted before, the problem of the imprecise estimates remains.

If we therefore return to eq. (10) and follow the remark on these relationships, the most straightforward solution seems to fix the parameter  $\nu$  from the outside. But at what level? It would in this respect be desirable to have some evidence, possibly by way of analogy, from the psychological literature. Here we are left with an *a priori* plausibility of  $\nu$  as the only criterion. Referring back to the transition probabilities in (4) and, for simplicity and just for the moment being, taking  $\phi_o = 0$  and  $x = 0$  to characterize a (hypothetical) neutral state, our most recent estimate  $\nu = 0.082$  has the immediate interpretation that on average an individual agent would autonomously switch every  $1/0.082 \approx 12$  months from pessimism to optimism or *vice versa*. This seems a reasonable order of magnitude given the kind of expectations the agents have to form, and one might even wonder if a higher or a lower flexibility would be more plausible.<sup>22</sup>

Nevertheless, before contenting ourselves with this solution and its remaining arbitrariness, let us consider the other parameters in the structural model. Setting  $\phi_y$ ,  $\phi_{\Delta y}$  and  $\phi_{\Delta x}$  at some exogenous value would be even more arbitrary, while (10) suggests that putting them equal to zero would deteriorate the fit too much, since  $\gamma_y$ ,  $\gamma_{\Delta y}$  and  $\beta_{1+\tau_x} = \beta_2$  are all significantly different from zero (cf. row 6 in Table 2). The estimations in rows 4–6 in Table 3 fully confirm this; apart from the fact that the outcome is of no great help for a more precise estimate of  $\nu$ , either.

---

<sup>22</sup>By fixing  $\nu = 0.082$  in the estimation, the four coefficients  $\phi_x$ ,  $\phi_{\Delta x}$ ,  $\phi_y$ ,  $\phi_{\Delta y}$  do turn significant, as shown by the standard errors in square brackets in the third row of Table 3. In particular, the *t*-ratios of  $\phi_y$  and  $\phi_{\Delta y}$  (the estimated coefficients divided by their standard error) are very similar to those of the coefficients  $\gamma_y$  and  $\gamma_{\Delta y}$  in the linear regression (9) (see the rows in bold face in Table 2). This only underscores the validity of the relationship disclosed by (10).

The last coefficient available is the coefficient  $\phi_x$  that represents the majority effect in the herding mechanism. Again, we have no direct clue for a sensible non-zero level. Furthermore, since it has always been a central parameter in the models in the literature and is, so to speak, the coefficient with which it all has begun (actually, the first discussions of this kind of theory included only  $\nu$  and  $\phi_x$  as non-zero coefficients), there is also a strong psychological barrier to let  $\phi_x$  disappear. Equation (10), however, would allow us to do so. With the estimates from the linear regression the first relationship in (10) would read  $1 - 2\nu \approx 1 - \nu c_2 \approx \beta_1 + \beta_{1+\tau_x} = \beta_1 + \beta_2 = 1.37 - 0.48$ . Solving it for  $\nu$  yields  $\nu \approx 0.055$ , which is a presentable order of magnitude as well: here autonomous switches of an agent would be expected to occur every  $1/0.055 \approx 18$  months.

The last row in the upper part of Table 3 shows the other coefficients and the log-likelihood resulting from this equation. First, all of the remaining coefficients are now highly significant. And second, if the likelihood is compared to that in the third row, then the deterioration in the fit is insignificant. Hence, in terms of parsimony, row 7 is to be preferred to row 3 (with  $\nu$  as part of the estimation). In other words, if we start from the estimation in row 7 and then consider to introduce  $\phi_x$  as an additional coefficient, the insufficient increase of the likelihood in row 3 would advise us against this generalization of the model. In this sense the estimation in row 7 is optimal, which we emphasize by the bold face characters.

To sum up, in the estimation of the structural model (5) – (7) on the ZEW expectations index we, legitimately, decide against the majority effect and dismiss it from the model. Herding is therefore exclusively represented by what we have called the moving-flock effect ( $\phi_{\Delta x} > 0$ ), and in combination with the hetero-reference mechanism we settle down on the estimates presented in row 7 of Table 3.

The estimation of the ifo expectations can proceed along the same lines. The lower part of Table 3 can thus be limited to the key results. The first row is the unconstrained estimation (except for subjecting the predisposition parameter to the consistency condition (13),  $\phi_o = \phi_o^e(\phi_x, \bar{x})$ , where, as it should be,  $\phi_o$  comes out negative). As opposed to what we obtained for the ZEW expectations, here the likelihood exceeds that of the corresponding linear regression (see row 4 in Table 2); but again the difference is not significant.

Of course, the problem of imprecise parameter estimates does not disappear. It is, however, remarkable that now one of the coefficients is significant. Moreover, this is just the

coefficient that we have decided to discard, namely  $\phi_x$  (see the standard error in bold face). The second row reveals that this significance is spurious. As in the estimations before, omitting  $\phi_x$  does not significantly lower the goodness-of-fit. A slight difference from the ZEW results is that the coefficient  $\phi_y$  on the levels of the output gap remains insignificant. Excluding it from the model yields the estimation in the last row of the table, which we present as our upshot for the ifo expectations index.

We are thus in a position to ask for the similarities and dissimilarities in the expectation formation by the agents in the two panels of ZEW and ifo. Four observations can be made in this regard. First, the ZEW agents have a predisposition toward optimism, the ifo agents toward pessimism. Our model captures this tendency by, respectively, positive and negative estimates of the parameter  $\phi_o$  (*via* the parameter  $\phi_x$  in (13) as well as when  $\phi_o$  is estimated directly). While one would not need a theoretical model for such a conclusion, this does not lessen its significance. In contrast, our second conclusion which refers to the agents' general flexibility would not be possible without a theoretical framework. Here we find, on the basis of our thought experiment of autonomous switches from pessimism to optimism and *vice versa* in a hypothetical state of equilibrium, that the ifo agents are more "flexible". They would on average switch every  $1/0.081 \approx 12$  months, whereas the expected frequency of the ZEW agents is 18 months (as noted above).

The third point concerns the agents' responsiveness to economic activity in the model's hetero-reference component. Although our estimation upshots show some differences in the coefficients  $\phi_y$  and  $\phi_{\Delta y}$ , they should not be overrated. On the one hand, the two coefficients  $\phi_{\Delta y}$  are almost the same if we compare the ZEW estimates with row 2 for the ifo expectations. On the other hand, re-estimating the ZEW expectations under the constraint  $\phi_y = 0$  leads to  $\phi_{\Delta y} = 34.4$ , while the moving-flock coefficient increases to (only)  $\phi_{\Delta x} = 4.85$  (though the fit is substantially poorer). The fourth observation concerns the herding mechanism. After we have confined it to the moving-flock effect, which has been carefully justified, we see that the two groups of agents are very similar in this behavioural characteristic. Actually, the two estimates of the coefficient  $\phi_{\Delta x}$  (3.86 and 4.27) can hardly be told apart.

In a very succinct way and at the risk of oversimplification we may thus summarize the estimations of our basic model as follows. Apart from a stronger predisposition toward pessimism and a somewhat higher, as we have called it, flexibility on the part of the ifo

agents, the two panels of ZEW and ifo agents are not markedly different: they share the same herding mechanism and react in similar ways to the arrival of new information.

## 4 Extensions of the basic specification

For the feedbacks on the agents' transition probabilities one can certainly think of many additional effects that might be worth exploring. We limit ourselves to two extensions of the model. First we ask a question that is obvious as soon as we have the notion of two herds the agents may be following, namely, if there are also effects from one herd (or flock) to the other. More technically, we study possible cross influences of the two climate indices. The second extension of the model introduces a variable that is possibly taken into account by the agents but remains unobservable to the researcher.

### 4.1 Cross effects between the ZEW and ifo panel

The concept of the herd, flock or crowd has so far assumed that regarding their state of mind the agents only communicate within their own well specified group. The possibility that, besides the fundamental data, the ZEW agents may also pay attention to the sentiment of the ifo agents, and *vice versa*, is thus neglected.

Instead of uniting the two flocks of ZEW and ifo agents to form one homogeneous group, we continue to keep the two panels apart but now admit cross effects from one climate index on the other. A first indication that this might improve the performance of the model is given by the linear regressions conducted by Lahl and Hüfner (2003). They report a significant *t*-statistic for the ifo index if one lag of it is added to their autoregressive equation for the ZEW index.<sup>23</sup> In the following we want to investigate this extension of one index possibly impacting on the other in a more systematic way, and in both directions.

The extension of the structural model (5)–(7) is straightforward. If  $x$  continues to be the estimated index and  $\xi$  is now introduced to refer to the alternative “outside” index, we only have to add  $\xi_{t-1}$  and, for a suitable lag  $\tau_\xi$ , its rate of change  $\Delta_{\tau_\xi}\xi_{t-1}$  in the specification

---

<sup>23</sup>Incidentally, including instead the first differences of the (nonstationary) US Consumer Confidence showed a similar effect.

of the feedback index  $f_{t-1}$ . The estimation equation then reads,

$$x_t = x_{t-1} + \nu [(1-x_{t-1}) \exp(f_t) - (1+x_{t-1}) \exp(-f_t)] + \varepsilon_{x,t} \quad (14)$$

$$\begin{aligned} f_{t-1} &= \phi_o + \phi_x x_{t-1} + \phi_{\Delta x} \Delta_{\tau_x} x_{t-1} + \phi_y y_{t-1} + \phi_{\Delta y} \Delta_{\tau_y} y_{t-1} + \phi_{\xi} \xi_{t-1} + \phi_{\Delta \xi} \Delta_{\tau_{\xi}} \xi_{t-1} \\ \phi_o &= \phi_o^e(\phi_x, \bar{x}) \text{ from (13)} \end{aligned} \quad (15)$$

Similar to the preparation of the results in Table 3 for the basic model, we have indeed checked that, if at all, only two lags of the alternative index are significant, the first one being  $t-1$ . Equation (16) collects the optimal lags in the rates of change of  $\xi$  (which yield the highest likelihood), together with those established for the other variables:

$$\begin{array}{llll} \tau_x = 1 & \tau_y = 4 & \tau_{\xi} = 2 & \text{for } x = \text{ZEW index} \\ \tau_x = 3 & \tau_y = 6 & \tau_{\xi} = 1 & \text{for } x = \text{ifo index} \end{array} \quad (16)$$

Table 4 presents the main estimations of eqs (14)–(16). To begin with the ZEW index, the first row in the upper part of the table reproduces row 3 from Table 3. Its likelihood serves as a benchmark to assess whether the ifo agents have a significant influence on the expectations formed by the ZEW agents, through what direct or indirect channels so ever.

The second row of the table replaces the feedback from the output gap with the feedback from the ifo index. With respect to the goodness-of-fit they bring about, the two effects are about equally strong. Combining the two effects in the third row shows that each of the two effects accomplishes a significant improvement over the isolated contribution of the other.

This characterization does not yet take the precision of the single coefficients into account. Precision is again obtained by discarding the majority effect in the herding mechanism, setting  $\phi_x = 0$ . For a better comparison with the basic model, the fourth row in the table repeats the estimation on which we have settled down in row 7 of Table 3, where all of the coefficients have come out highly significant. This feature is fully maintained in row 5 of Table 4 if the changes  $\Delta_{\tau_{\xi}} \xi_{t-1}$  of the ifo climate are included as the only additional feedback variable. The coefficient  $\phi_{\Delta \xi}$  on the latter proves to be highly significant, too, while, as already indicated by the high standard error in row 3, the level effect from  $\xi_{t-1}$  would be rather insubstantial. Hence, again, the changes in the agents' attitudes are more important than their current level. Since also the likelihood increases considerably from row 4 to row 5, we can conclude that the augmented model (14)–(16) is strongly supported by



	$\nu$	$\phi_x$	$\phi_{\Delta x}$	$\phi_y$	$\phi_{\Delta y}$	$\phi_\xi$	$\phi_{\Delta \xi}$	2 L	RMSE
<u>ZEW expectations</u>									
1	0.082 (0.054)	0.41 (0.45)	2.83 (1.59)	-6.23 (4.27)	37.3 (24.6)	---	---	384.8	0.0793
2	0.105 (0.070)	0.71 (0.34)	1.66 (1.08)	---	---	-0.19 (0.17)	2.67 (1.77)	386.2	0.0790
3	0.088 (0.062)	0.42 (0.54)	1.94 (1.25)	-3.49 (3.45)	29.5 (21.5)	-0.15 (0.21)	2.64 (1.84)	394.7	0.0770
4	0.053 (0.011)	---	3.86 (0.74)	-9.05 (3.05)	57.7 (14.0)	---	---	383.7	0.0795
5	<b>0.062</b> (0.011)	---	<b>2.59</b> (0.66)	<b>-5.65</b> (2.71)	<b>42.5</b> (12.3)	---	<b>3.47</b> (1.09)	<b>393.9</b>	<b>0.0772</b>
<u>ifo expectations</u>									
1	0.196 (0.127)	0.68 (0.26)	1.49 (1.01)	-1.68 (1.36)	20.5 (13.8)	---	---	462.2	0.0633
2	0.122 (0.142)	0.64 (0.54)	2.22 (2.58)	---	---	0.05 (0.10)	0.37 (0.50)	448.6	0.0659
3	0.200 (0.136)	0.73 (0.27)	1.20 (0.86)	-1.70 (1.35)	20.0 (13.8)	-0.00 (0.04)	0.22 (0.21)	464.1	0.0630
4	<b>0.081</b> (0.015)	---	<b>4.27</b> (0.82)	---	<b>39.9</b> (8.9)	---	---	<b>457.1</b>	<b>0.0643</b>
5	0.075 (0.016)	---	3.94 (0.88)	---	42.2 (9.7)	---	0.54 (0.45)	458.8	0.0639

**Table 4:** Estimations of cross effects, eqs (14) – (16).

the data and, with the variables selected, provides a powerful description of the expectation formation process of the ZEW agents.

Adding the ifo climate to the variables of the basic model is quite likely to change the original coefficients. Comparing row 4 and 5, as well the imprecise estimates in row 1 and 3, it is, however, seen that these changes follow a consistent pattern; namely, the coefficients  $\phi_{\Delta x}$ ,  $\phi_y$ ,  $\phi_{\Delta y}$  are all moderately lowered through the new variable(s), while  $\phi_x$  remains unaffected. The flexibility parameter  $\nu$  should not, and does not, change significantly. For the basic model it can thus be said that the outside ifo index is an important omitted variable, whose influence is quite regularly distributed across the reaction parameters it has estimated.

The lower part of Table 4 documents the influence of the ZEW index on the ifo index, or rather its non-existing influence. The first row reproduces the first row of the ifo estimations in Table 3. In row 2 and 3 it is seen that neither replacing the output gap nor combining it with the ifo index is of any relevance. The same conclusion has to be drawn if the estimations are restricted to the most significant coefficients. Hence also in the extended framework it turns out that for the ifo index the estimation in row 3 of Table 3, which is repeated in row 4 of Table 4, remains the result that we can offer to take home. Of course, it cannot be ruled out that there might be other variables that have a greater impact on the ifo expectations. Considering, however, the relatively low RMSE and the corresponding  $R^2$ , which amounts to 0.922, the explanatory power of this estimation is already remarkable.

## 4.2 Estimation with an unobservable variable

In this section we try an estimation approach that is very different from the previous linear or nonlinear least-squares minimization. In principle, the approach can account for the effects that so far have been omitted. It does this by lumping them all together in one single variable that is added to our feedback index  $f$ . That is, the transition probabilities of the agents will vary with the sum of the index  $f$  and this new variable, which is stochastic as well as unobservable. The promise of “in principle” is qualified by the need to specify a stochastic law of motion for that variable, whose parameters will be part of the estimation. To guard against the possible criticism of arbitrariness, which to deal with would require a careful investigation of a battery of alternative cases, we will content ourselves here with a most parsimonious form.

Let us denote the unobservable variable by  $a_t$ . We assume that it follows a first-order autoregressive process with an autocorrelation coefficient  $\rho$ . Let  $\sigma_a$  and  $\sigma_x$  be the standard deviations of the two random terms that impact on  $a_t$  and the climate index  $x_t$ , respectively, and reserve the notation  $\eta_a$ ,  $\eta_x$  to (independent) draws from the standard normal distribution (with mean zero and variance one). The system to be estimated then reads,

$$x_t = x_{t-1} + \nu [(1-x_{t-1}) \exp(f_{t-1}+a_t) - (1+x_{t-1}) \exp(-f_{t-1}-a_t)] + \sigma_x \eta_{x,t} \quad (17)$$

$$a_t = \rho a_{t-1} + \sigma_a \eta_{a,t} \quad (18)$$

$$\begin{aligned}
f_{t-1} &= \phi_o + \phi_x x_{t-1} + \phi_{\Delta x} \Delta_{\tau_x} x_{t-1} + \phi_y y_{t-1} + \phi_{\Delta y} \Delta_{\tau_y} y_{t-1} + \phi_\xi \xi_{t-1} + \phi_{\Delta \xi} \Delta_{\tau_\xi} \xi_{t-1} \quad (19) \\
\phi_o &= \phi_o^e(\phi_x, \bar{x}) \text{ from (13) , } \quad \text{the lags } \tau_x, \tau_y, \tau_\xi \text{ from (16)}
\end{aligned}$$

The variable  $a_t$  can be conceived of as representing a general composite variable made up of additional fundamental data such as wages, interest rates, exchange rates, political news, etc., or it may (alternatively or additionally) comprise the measurement errors that we have discussed when introducing the basic equations (5)–(7). It is, however, presupposed that this “catch-all” variable evolves in a fairly regular manner.

An estimation of model (17)–(19) uses the Kalman filter to set up a likelihood function, which is to be maximized. Under the assumption of normal distributions, the Kalman filter is concerned with the prior and posterior probability densities of the unobservable state in each period  $t$ , that is, with their means and variances. In essence, the Kalman filter is an optimal updating algorithm for them, given the parameters in the equations (which include the standard deviations  $\sigma_x$  and  $\sigma_a$ ). Since the procedure is based on linear relationships but (17) is nonlinear, one has to work with linear approximations. They constitute the extended Kalman filter, as it is more precisely called.<sup>24</sup>

Relying on the period- $t$  prior estimate of the unobservable variable  $a_t$ , which uses information up to period  $t-1$  and is denoted by  $a_{t|t-1}$ , one can predict the value  $x_{t|t-1}$  of the observable index  $x_t$  from the right-hand side of (17) (by dropping  $\eta_{x,t}$  and substituting  $a_{t|t-1}$  for  $a_t$ ). The prediction error  $e_{t|t-1}$  is the difference between  $x_{t|t-1}$  and the value  $x_t$  actually observed. Together with a variance that is equally part of the updating mechanism, one can then compute the period- $t$  probability associated with  $e_{t|t-1}$ . The sum (over  $t$ ) of the logs of these probabilities yields the value  $L$  of the log-likelihood function. Of course, the sequence of the prediction errors changes with the parameters in (17)–(19), and so does  $L$ . Estimation of (17)–(19) means to find the set of parameters that maximizes the value of the thus defined log-likelihood  $L$ . The details behind this short summary are given in Appendix 3.

It is important to note that the degenerate case of this likelihood maximization, which is obtained by fixing  $a_t=0$ ,  $\sigma_a=0$  in (18), is equivalent to the nonlinear least-squares

---

<sup>24</sup>There are more elaborated approaches to deal with nonlinearities in this setting; in particular, the “unscented Kalman filter”. Besides the (substantial) higher costs of computation, these procedures would require a higher effort on the part of the reader either to believe in the final results or to understand what the algorithms are about.

estimation from above. This, in particular, implies that in order to evaluate the goodness-of-fit of the different methods, the likelihood values reported in the previous tables can be directly compared with the values resulting from the estimation of (17)–(19).

	$\nu$	$\phi_x$	$\phi_{\Delta x}$	$\phi_y$	$\phi_{\Delta y}$	$\phi_{\Delta \xi}$	$\rho$	$\sigma_a$	$\sigma_x$	2L
1	0.053 (0.011)	—	3.86 (0.74)	−9.05 (3.05)	57.7 (14.0)	—	—	—	0.0793	383.7
2	0.058 (0.007)	—	2.84 (1.11)	−10.47 (3.09)	61.5 (13.9)	—	0.15 (0.08)	0.672 (0.140)	0.0001	394.3
3	<b>0.055</b> (0.006)	—	<b>3.84</b> (0.59)	<b>−9.91</b> (2.81)	<b>64.5</b> (13.2)	—	—	<b>0.705</b> (0.076)	<b>0.0000</b>	<b>393.1</b>
4	0.072 (0.010)	0.34 <b>(0.18)</b>	3.16 (0.55)	−7.90 (2.46)	47.6 (13.5)	—	—	0.560 (0.070)	0.0000	394.4
5	0.062 (0.011)	—	2.59 (0.66)	−5.65 (2.71)	42.5 (12.3)	3.47 (1.09)	—	—	0.0772	393.9
6	<b>0.059</b> (0.006)	—	<b>2.62</b> (0.57)	<b>−6.68</b> (2.64)	<b>50.8</b> (12.5)	<b>4.40</b> (0.07)	—	<b>0.625</b> (0.068)	<b>0.0000</b>	<b>409.5</b>
7	0.061 (0.008)	0.06 <b>(0.20)</b>	2.56 (0.57)	−6.46 (2.61)	48.3 (14.3)	4.20 (0.07)	—	0.605 (0.071)	0.0000	409.5

**Table 5:** Kalman filter estimations of (17)–(19) for the ZEW index.

The question of whether an estimation of (17)–(19) can outperform the previous results has two clear answers: ‘no’ for the ifo index, and ‘yes’ for the ZEW index. We abstain from a documentation of all the failures when applying (17)–(19) to the ifo index and concentrate on the main results for the ZEW index. They are reported in Table 5, where in two blocks the estimations are carried out without and with incorporating the cross effects from the alternative index, i.e., from the ifo index. To begin with the simpler specification, the first row reproduces our upshot of the estimation of the basic model, row 7 from Table 3. Freezing  $a_t=0$  in (18), we get a standard deviation  $\sigma_x = 0.0795$  in eq. (17) and, as just asserted, the same likelihood as in the NLS estimation.

The second row in Table 5 shows that the introduction of the unobserved variable  $a_t$  leads to a marked improvement. The serial correlation in  $a_t$  is quite low and cancelling it,  $\rho = 0$ , does not cause a significant deterioration in the likelihood. The bold face figures

in the third row of the table are thus the optimal result that we get for the ZEW climate, when the feedbacks in the transition probabilities are limited to the index itself and the output gap.

Comparing row 3 to row 1, it is seen that the coefficient estimates do not differ very much. The standard errors for  $\phi_{\Delta x}$ ,  $\phi_y$ ,  $\phi_{\Delta y}$  are slightly better in the new version, while the precision of the flexibility coefficient  $\nu$  has almost doubled. These positive features have become possible by a changing role of the random perturbations. With  $\sigma_x \approx 0$  they no longer show up as an additive term to the climate's aggregate adjustment equation, where they summarize influences from outside the model and perhaps also correct for possible misspecifications in the functional form. Instead, the perturbations are directly connected to the feedback index, which means they can be regarded as occurring within the modelling framework. It is furthermore remarkable that they are serially uncorrelated. Hence if there are important variables omitted by us, they at least do not behave too regularly. These observations underline the explanatory power of the model's transition probabilities as they are set out, and our choice of the variables specifying the feedback index.

Technically, the changing place where the random forces take effect indicates the benefits from the nonlinear structure of the model. It appears that the superior results from the estimation of (17)–(19) can be ascribed to the curvature, however slight, in the exponential function, and the variability of the unobserved perturbations  $a_t$  (brought about by  $\sigma_a \eta_{a,t}$ ) is able to exploit this feature.

Things are very similar when the variable  $\Delta_{\tau_\xi} \xi_{t-1}$  enters the feedback index in addition. The fifth row in Table 5 reiterates the final result from Table 4, and row 6 in Table 5 points out the improvement achieved by (17)–(19), which, in terms of the likelihood, is even stronger than before. Particularly astonishing is the minimal standard error of the estimate of  $\phi_{\Delta \xi}$ . Again, the function of the random perturbations of  $x_t$  is completely taken over by the random forces impacting directly on the transition probabilities, and again they do not exhibit significant autocorrelation.

Finally, we come back to the majority effect in the herding mechanism and the corresponding coefficient  $\phi_x$ . Could it be that in the new approach, which can better exploit the model's nonlinearities, this effect has a greater role to play? The estimations in row 4 and 7 of Table 5 disprove this idea, as in both cases the increase in the likelihood is far from significant (despite the relatively low standard error of  $\phi_x$  in row 4). The validity of

the generalized model (17)–(19) is furthermore accentuated by the fact that now, although with the reintroduction of  $\phi_x$  we could have “one parameter too many” as discussed in Section 3, the estimates of the other coefficients maintain their precision.

To sum up, the emphasized rows 3 and 6 in Table 5 are indeed an adequate description of how the agents in the ZEW panel form their optimistic and pessimistic expectations about the economy, qualitatively (regarding especially the choice of the selected variables) as well as quantitatively (regarding the reaction intensities). In the estimations of the model for the ifo agents, the exogenous random forces have turned out to play a different role, since here they are disconnected from the feedback index and act on the aggregate index  $x_t$  directly. The good estimation results (row 3 in Table 3 and row 4 in Table 4) show that nevertheless this panel, too, provides sound support for our modelling approach.<sup>25</sup>

## 5 Conclusion

The paper has designed a population of agents taking one of two opposite attitudes like, for example, optimism and pessimism. From a macroscopic point of view, their switches from one attitude to the other are most suitably modelled by transition probabilities. In this way an adjustment equation for the aggregate climate in the population can be derived, whose stochastic diffusion elements become negligible if the population is large. Various feedbacks can be incorporated into this dynamic equation by specifying the transition probabilities as functions of (a) the current level of the climate index, (b) its rate of change, and (c) other fundamental data. Feedbacks (a) and (b) can be said to constitute a herding dynamics, while (c) allows the agents to take external norms into account that may have an additional influence on their attitudes.

We would like to present our approach as an alternative to the mainstream macroeconomics founded on the representative agent. It starts out from heterogeneous agents who constantly change their state of mind and remain heterogeneous. Their aggregate opinion,

---

<sup>25</sup>One obvious reason why the approach (17)–(19) does not work for the ifo agents could be that the specification of the stochastic law for the unobservable variable is too simple, and that a broader ARMA specification might be more successful. Alternatively, one could think of a respecification of the functional form of the transition probabilities, although there is no other candidate that would equally be as elegant as the assumption of linearity in their relative changes. In the interest of a uniform formulation of the model, it does not seem very worthwhile to set about working in this direction.

or climate, can nevertheless be conveniently studied at the macroeconomic level, like in any other heterodox and feedback-guided modelling framework. Since an explicit micro-economic basis has been provided, we do not need to invoke a “typical” agent to whom also heterodox economists (explicitly or implicitly) refer to discuss behavioural patterns.

Postulating feedbacks from appropriate macroeconomic fundamentals like aggregate output, wages or interest rates, the climate adjustment equation can be easily combined with other building blocks from the toolbox of feedback-guided macro modelling. This has been shown elsewhere in a parsimonious theoretical model of the economy, in Franke (2007). It develops two- and three-dimensional Goodwinian income distribution dynamics in which one can legitimately speak of “animal spirits” determining investment demand of firms and thus economic activity. By contrast, the present paper is concerned with the empirical validity of our modelling design. While in the aforementioned macro dynamics the climate index is an unobserved variable, one can alternatively use survey expectations as a proxy for it, so that the law that governs the changes in the climate can be estimated directly. In Germany there are monthly data on two such surveys that are well suited for this task, whose indices are constructed by the ifo and ZEW institute, respectively. They derive from interviews with people from the business and financial world, where quite in accordance with our two-state setting the agents only have three options to summarize their expectations about the general business situation over the next six months, namely, better, worse and about the same.

Choosing the output gap and its rate of change as the agents’ fundamental news and applying the thus specified model to this kind of data proved rather successful. The estimated coefficients made economic sense and were significant or even highly significant. Most importantly, we could identify a significant herding mechanism, which is of roughly equal intensity in the two panels. It is in this respect remarkable that the individual agents tend to change their attitude not so much in response to the current level of the majority opinion, but in response to its most recent changes. That is, we can state that in both panels the formation of expectations is characterized by strong herding, in that figuratively speaking the agents do not just join the crowd but follow each single motion of the crowd. An additional finding is that the ZEW agents from the financial sector are also influenced by the motions of the “crowd” of the ifo agents, who are leaders and senior managers from

the business sector, whereas there are no significant cross effects in the other direction. All these results demonstrate a substantial explanatory power of our theoretical model.

It would, of course, be desirable to test the approach with other similarly constructed survey data on the one hand, and on the other hand to estimate an entire macroeconomic system with our aggregate adjustment equation as a constituent part, where the climate index itself remains unobserved. It is furthermore straightforward to extend the model to three states for the agents' attitude: optimistic, pessimistic and indifferent, say. If in addition to the difference between optimistic and pessimistic agents also data on the share of indifferent agents are available, one could try an even harder test of the microfounded modelling approach here put forward.

## A Appendices

### A.1 A critical benchmark of the majority effect

Motivated by the results that are readily obtained from a simplified version of the model, the parameter  $\phi_x$  can be used to distinguish between a weak and a strong majority effect. Setting all other coefficients except  $\nu$  equal to zero and, for the moment being, considering the model's deterministic and continuous-time version as discussed at the end of Section 2.2, eqs (5), (6) boil down to a one-dimensional differential equation,

$$\dot{x} = \nu [(1-x) \exp(\phi_x x) - (1+x) \exp(-\phi_x x)] \quad (20)$$

Clearly,  $x^* = 0$  is a stationary point of this pure herding dynamics. It is globally stable if  $0 < \phi_x < 1$ , and it is unstable if  $\phi_x > 1$  (for this and the following see, e.g., Lux, 1995, pp. 885f). In the latter case, the process of optimism feeding optimism (or pessimism feeding pessimism) is persistent and the general climate is eventually locked in a state where a constant majority of agents is optimistic (or pessimistic, respectively). More exactly, the dynamics converges to a uniquely determined optimistic climate  $x^{opt} > 0$  if it has started out from an initially optimistic climate  $x_0 > 0$ , and it converges to a uniquely determined pessimistic climate  $x^{pess} < 0$  if  $x_0 < 0$  at the start. Under the alternative condition  $0 < \phi_x < 1$ , it is low, but not strong, optimism that feeds optimism, such that the dynamics eventually reaches the balancing state  $x^* = 0$  if the initial level  $x_0$  is negative (and conversely low pessimism feeding pessimism when  $x_0 > 0$ ). Therefore, one may speak



of weak herding in connection with the majority effect if  $0 < \phi_x < 1$ , and of strong herding in this respect if  $\phi_x > 1$  is prevailing.

The critical value  $\phi_x^c = 1$  dividing stability from instability would be modified if also  $\phi_{\Delta x}$  were nonzero (and  $\Delta_{\tau_x} x_{t-1}$  replaced with the derivative  $\dot{x}$ , in which case (20) would be an implicit differential equation); or if  $\phi_y \neq 0$ ,  $\phi_{\Delta y} \neq 0$  and the adjustment equation for the climate were combined with some other component determining the motions of  $y$ . However, for a rough-and-ready characterization of a weak and strong majority effect, which is independent of further modelling assumptions,  $\phi_x^c = 1$  is a good enough reference value.

## A.2 Side calculations

We want to relate the linearized version of the structural model (5) to the linear regression equation (9). We begin with the observation following from (13) that

$$\bar{f} := \phi_o^e + \phi_x \bar{x} = -\frac{1}{2} \ln \frac{1-\bar{x}}{1+\bar{x}} = \ln \left[ \left( \frac{1-\bar{x}}{1+\bar{x}} \right)^{-1/2} \right] = \ln[\sqrt{1+\bar{x}}/\sqrt{1-\bar{x}}] \quad (21)$$

Hence, taking logs,

$$\begin{aligned} c_1 &:= (1-\bar{x}) \exp(\bar{f}) + (1+\bar{x}) \exp(-\bar{f}) = 2\sqrt{1+\bar{x}}\sqrt{1-\bar{x}} \\ c_2 &:= \exp(\bar{f}) + \exp(-\bar{f}) = 2/[\sqrt{1+\bar{x}}\sqrt{1-\bar{x}}] \end{aligned}$$

Denote the right-hand side of eq. (5) by  $G$ , which dropping the time indices is a function of  $x, \Delta x, y, \Delta y$ . Supposing  $\phi_o = \phi_o^e$  and partially differentiating  $G$  with respect to  $x$ , evaluated at the stationary point  $(x, \Delta x, y, \Delta y) = (\bar{x}, 0, 0, 0)$ , gives

$$\begin{aligned} \partial G / \partial x &= 1 + \nu [(1-\bar{x}) \exp(\bar{f}) \phi_x - \exp(\bar{f}) + (1+\bar{x}) \exp(-\bar{f}) \phi_x - \exp(-\bar{f})] \\ &= 1 + \nu [c_1 \phi_x - c_2] \end{aligned}$$

To obtain the corresponding coefficient in (9) it has to be noted that, with respect to the selected lag,  $\beta_1 x_{t-1} + \beta_\theta x_{t-\theta} = \beta_1 x_{t-1} + \beta_{1+\tau_x} x_{t-1-\tau_x}$  in the notation of (5), (6). Rewriting this sum as  $(\beta_1 + \beta_{1+\tau_x}) x_{t-1} - \beta_{1+\tau_x} (x_{t-1} - x_{t-1-\tau_x}) = (\beta_1 + \beta_{1+\tau_x}) x_{t-1} - \tau_x \beta_{1+\tau_x} \Delta_{\tau_x} x_{t-1}$  establishes the first approximate equality in (10).

The partial derivatives of the function  $G$  with respect to the other variables all have the same structure. For  $u = \Delta x, y, \Delta y$  we have

$$\partial G / \partial u = \nu [(1-\bar{x}) \exp(\bar{f}) \phi_u + (1+\bar{x}) \exp(-\bar{f}) \phi_u] = \nu c_1 \phi_u$$

The correspondence of the coefficients  $\phi_u$  in (5), (6) and  $-\tau_x \beta_{1+\tau_x}$ ,  $\gamma_y$ ,  $\gamma_{\Delta y}$  from (9) is then obvious, which completes the proof of (10).

### A.3 Estimation with the extended Kalman filter

To estimate (18)–(19), first simplify notation by abbreviating  $z_t = (\Delta_{\tau_x} x_t, y_t, \Delta_{\tau_y} y_t)$  and summarizing the two equations (17), (19) as

$$x_t = F(x_{t-1}, a_t, z_{t-1}) + \sigma_x \eta_{x,t} \quad (22)$$

Empirical data of  $x_t$  and  $z_t$  are available from  $t = 0$  to  $T$ , so that  $t = 1, \dots, T$  in these equations.

Central to the Kalman filter are the prior and posterior probability densities of the unobservable state  $a_t$ . Putting  $X_t = \{x_t, z_t, x_{t-1}, z_{t-1}, \dots, x_1, z_1\}$ , the mean and variance of these densities are denoted as

$$\begin{aligned} a_{t|t-1} &:= E(a_t | X_{t-1}) \\ P_{t|t-1} &:= \text{Var}(a_t | X_{t-1}) = E[(a_t - a_{t|t-1})^2] \\ a_{t|t} &:= E(a_t | X_t) \\ P_{t|t} &:= \text{Var}(a_t | X_t) = E[(a_t - a_{t|t})^2] \end{aligned} \quad (23)$$

The Kalman filter is an algorithm determining  $a_{t|t-1}$ ,  $P_{t|t-1}$ , etc., that can be easily iterated forward once initial estimates of the state and the corresponding variance,  $P_{0|0}$  and  $a_{0|0}$ , are specified.<sup>26</sup> Following the advice in Section 3.3.4 of Harvey (1989), we put

$$a_{0|0} = 0 \quad P_{0|0} = \sigma_a^2 / (1 - \rho) \quad (24)$$

Being in period  $t \geq 1$  the first step is the estimation of the state  $a_t$  and its variance on the basis of the information up to  $t-1$ . The resulting equations (25) and (26) are the so-called prediction equations:

$$a_{t|t-1} = \rho a_{t-1|t-1} \quad (25)$$

$$\begin{aligned} P_{t|t-1} &= E[(a_t - a_{t|t-1})^2] = E[(\rho a_{t-1} + \sigma_a \eta_{a,t} - \rho a_{t-1|t-1})^2] \\ &= \rho^2 E[(a_{t-1} - a_{t-1|t-1})^2] + \sigma_a^2 \\ &= \rho^2 P_{t-1|t-1} + \sigma_a^2 \end{aligned} \quad (26)$$

---

<sup>26</sup>The following presentation draws on a short outline of the (linear) Kalman filter by Ellen L. Hamaker from the University of Utrecht, “Kalman filter algorithm”, which is contained in the FORTRAN package provided by Conor V. Dolan (University of Amsterdam) on his homepage, <http://users.fmg.uva.nl/cdolan>.

The corresponding prior estimate of the observed variable  $x_t$  and the estimation error, which are denoted by  $x_{t|t-1}$  and  $e_{t|t-1}$ , respectively, are

$$x_{t|t-1} = F(x_{t-1}, a_{t|t-1}, z_{t-1}) \quad (27)$$

$$e_{t|t-1} = x_t - x_{t|t-1} \quad (28)$$

While the moments of  $e_{t|t-1}$  will in general be difficult to compute if  $F$  is nonlinear in the state variable, they can be conveniently approximated if  $F$  is linearized in this respect. To this end we define

$$F_{a,t} := \partial F(x_{t-1}, a_{t|t-1}, z_{t-1}) / \partial a \quad (29)$$

$$\tilde{e}_{t|t-1} := F_{a,t}(a_t - a_{t|t-1}) + \sigma_x \eta_{x,t} \quad (30)$$

and note that  $\tilde{e}_{t|t-1}$  is a linear approximation of the estimation error:

$$\begin{aligned} \tilde{e}_{t|t-1} &= F(x_{t-1}, a_{t|t-1}, z_{t-1}) + F_{a,t}(a_t - a_{t|t-1}) + \sigma_x \eta_{x,t} - F(x_{t-1}, a_{t|t-1}, z_{t-1}) \\ &\approx F(x_{t-1}, a_t, z_{t-1}) + \sigma_x \eta_{x,t} - F(x_{t-1}, a_{t|t-1}, z_{t-1}) \\ &= x_t - x_{t|t-1} = e_{t|t-1} \end{aligned}$$

Moreover,  $\tilde{e}_{t|t-1}$  has mean zero. This is obvious for  $F_{a,t} = 0$ , and otherwise follows from (30) and  $E(\tilde{e}_{t|t-1})/F_{a,t} = E(a_t - a_{t|t-1}) = E(\rho a_{t-1} - \rho a_{t-1|t-1}) = E(\rho^{t-1} a_1 - \rho^{t-1} a_{1|1}) = 0$ . The variance of  $e_{t|t-1}$  is then approximated by  $H_t := E[(\tilde{e}_{t|t-1})^2]$  and we get

$$\begin{aligned} H_t &= E\{[F_{a,t}(a_t - a_{t|t-1}) + \sigma_x \eta_{x,t}]^2\} = F_{a,t}^2 E[(a_t - a_{t|t-1})^2] + \sigma_x^2 E[\eta_{x,t}^2] \\ &= F_{a,t}^2 P_{t|t-1} + \sigma_x^2 \end{aligned} \quad (31)$$

Below the covariance between the state  $a_t$  and the error  $e_{t|t-1}$  is needed, conditioned on the information up to  $t-1$ . Observing that  $\text{Cov}(a_t, e_{t|t-1} | X_{t-1}) = E[(a_t - E a_t)(e_{t|t-1} - E e_{t|t-1}) | X_{t-1}] \approx E[(a_t - a_{t|t-1}) \tilde{e}_{t|t-1}] = E\{(a_t - a_{t|t-1}) [F_{a,t}(a_t - a_{t|t-1}) + \sigma_x \eta_{x,t}]\} = F_{a,t} E[(a_t - a_{t|t-1})^2]$ , this covariance can be approximated by

$$J_t = F_{a,t} P_{t|t-1} \quad (32)$$

The next step has recourse to a basic theorem from statistical analysis (see, e.g., Durbin and Koopman, 2001, p. 37). Its general multivariate version says that if  $a, b, c$  are random vectors that are normally distributed with covariance matrices  $\Sigma_{ab}$ ,  $\Sigma_{ac}$ ,  $\Sigma_{cc}$  and the conditional covariance matrix  $\Sigma_{aa|b}$ , then

$$\begin{aligned} E(a | b, c) &= E(a | b) + \Sigma_{ac} \Sigma_{cc}^{-1} c \\ \text{Var}(a | b, c) &= \Sigma_{aa|b} - \Sigma_{ac} \Sigma_{cc}^{-1} \Sigma'_{ac} \end{aligned}$$

Of course, in the present simplified setting these are all real numbers. In order to apply the theorem, it is assumed that observation  $x_t$  as a random variable in the real world is normally distributed. By virtue of the first equation, the estimate of the state  $a_t$  can be updated as  $E(a_t | X_t) = E(a_t | X_{t-1}, e_{t|t-1}) = E(a_t | X_{t-1}) + \text{Cov}(a_t, e_{t|t-1}) \text{Var}(e_{t|t-1})^{-1} e_{t|t-1}$ . The latter product is approximately equal to  $J_t H_t^{-1} e_{t|t-1}$ . Thus, with definition (23), we put

$$a_{t|t} = a_{t|t-1} + J_t H_t^{-1} e_{t|t-1} \quad (33)$$

The second part of the theorem gives us  $\text{Var}(a_t | X_t) = \text{Var}(a_t | X_{t-1}, e_{t|t-1}) = \text{Var}(a_t | X_{t-1}) - \text{Cov}(a_t, e_{t|t-1})^2 \text{Var}(e_{t|t-1})^{-1}$ . On the basis of the same approximation as before we put

$$P_{t|t} = P_{t|t-1} - J_t^2 H_t^{-1} \quad (34)$$

Clearly, eqs (23)–(34) form a recursive system, which completes the description of the extended Kalman filter.

To estimate the parameters underlying (18)–(19), the output of the iteration algorithm enters the loglikelihood function. The function is given by

$$\begin{aligned} L(X_T) &:= \ln \text{prob}(x_2, \dots, x_T | x_1, z_1, \dots, z_T) \\ &= \sum_{t=1}^T \ln \text{prob}(x_t | X_{t-1}) \end{aligned} \quad (35)$$

where ‘prob’ denotes the probability density function and  $\text{prob}(x_1 | X_0) = \text{prob}(x_1 | a_{0|0}, P_{0|0})$ . Recalling that the random variable  $e_{t|t-1}$  has mean zero and variance  $H_t$ , and assuming that it is normally distributed, one has

$$\begin{aligned} \text{prob}(x_t | X_{t-1}) &= \text{prob}(x_t | x_{t-1}, \dots) = \text{prob}(e_{t|t-1}) \\ &= \frac{1}{\sqrt{2\pi H_t}} \exp \left[ -\frac{(e_{t|t-1})^2}{2 H_t} \right] \end{aligned}$$

Thus, the loglikelihood function reads,

$$L(X_T) = \frac{-T}{2} \ln 2\pi - \frac{1}{2} \sum_{t=1}^T \ln H_t - \frac{1}{2} \sum_{t=1}^T (e_{t|t-1})^2 / H_t \quad (36)$$

This function is to be maximized across  $\sigma_x$ ,  $\sigma_a$ ,  $\rho$  and the free parameters in the function  $F$ . The covariance matrix from which the standard errors of the estimates are derived is given by minus the inverse of the Hessian matrix of  $L(X_T)$  evaluated at the optimal solution (Davidson and MacKinnon, 2004, p. 415; the partial derivatives in the matrix are computed numerically).

As a byproduct, the filter allows us to evaluate the goodness-of-fit by referring to the one-period ahead predictions of the observed variable, which are the values of  $x_{t|t-1}$  in eq. (27). Note also that the procedure contains the NLS approach to eq. (22) (with  $a_t \equiv 0$ ) as a special case. In fact, with  $\sigma_a = 0$  it is easily seen that  $H_t = \sigma_x^2$  for all  $t$  in (36), and after concentrating this function with respect to  $\sigma_x^2$  (see, e.g., Davidson and MacKinnon, 2004, p.403), the maximization problem in (36) becomes one of minimizing the squared residuals  $\sum_t (e_{t|t-1})^2 = \sum_t [x_t - F(x_{t-1}, 0, z_{t-1})]^2$ .

## References

- ALFARANO, S. AND LUX, T. (2005), “A noise trader model as a generator of apparent power laws and long memory”, *Macroeconomic Dynamics* (forthcoming).
- ALFARANO, S., LUX, T. AND WAGNER, F. (2005), Estimation of agent-based models: The case of an asymmetric herding model”, *Computational Economics*, 26, 19–49.
- BROYER, S. AND SAVRY, G. (2002), “German leading indicators: Which one should be monitored?”, *CDC IXIS Capital Markets Flash*, No.2002–38; <http://www.zew.de/de/publikationen/Konjunkturerwartungen/fla2002-38gb.pdf>.
- DAVIDSON, R. AND MACKINNON, J.G. (2004), *Econometric Theory and Methods*. New York: Oxford University Press.
- DURBIN, J. AND KOOPMAN, S.J. (2001), *Time Series Analysis by State Space Methods*. New York: Oxford University Press.
- FLASCHEL, P., FRANKE, R. AND SEMMLER, W. (1997), *Dynamic Macroeconomics. Instability, Fluctuations, and Growth in Monetary Economies*. Cambridge, Mass: MIT Press.
- FRANKE, R. (2007), “Microfounded animal spirits and Goodwinian income distribution dynamics”, in P. Flaschel and M. Landesmann (eds.), *Effective Demand, Income Distribution and Growth. Research In Memory of the Work of Richard M. Goodwin*. London: Routledge (forthcoming).

- FRANKE, R. AND ASADA, T. (1994), “Expectations dynamics in a Keynes-Goodwin model of the business cycle”, *Journal of Economic Behavior and Organization*, 24, 273–295.
- FRANKE R. AND KALMBACH, P., “Structural change in the manufacturing sector and its impact on business-related services: An input-output study for Germany”, *Structural Change and Economic Dynamics*, 16, 467–488.
- HAAG, G., WEIDLICH, W. AND MENSCH, G. (1987), “The Schumpeter clock”, in D. Batten et al. (eds), *Economic Evolution and Structural Adjustment*. Berlin: Springer.
- HARVEY, A. (1989), *Forecasting, Structural Time Series Models and the Kalman Filter*. New York: Cambridge University Press.
- HÜFNER, F.P. AND SCHRÖEDER, M. (2002a), “Forecasting economic activity in Germany: How useful are sentiment indicators?”, ZEW Discussion Paper 02–56.
- HÜFNER, F.P. AND SCHRÖEDER, M. (2002b), “Prognosegehalt von ifo-Geschäftserwartungen und ZEW-Konjunkturerwartungen: Ein ökonometrischer Vergleich”, *Jahrbücher für Nationalökonomie und Statistik*, 223, 316–336.
- KIRMAN, A. (1993), “Ants, rationality, and recruitment”, *Quarterly Journal of Economics*, 108, 137–156.
- KRAFT, P., LANDES, T. AND WEISE, P. (1986), “Dynamic aspects of a stochastic business cycle model”, *Methods of Operations Research*, 53, 445–453.
- LAHL, D. AND HÜFNER, F. (2003), “What determines the ZEW indicator?”, ZEW Discussion Paper 03–48.
- LUX, T. (1995), “Herd behaviour, bubbles and crashes”, *Economic Journal*, 105, 881–896.
- LUX, T. (1997), “Time variation of second moments from a noise trader/infection model”, *Journal of Economic Dynamics and Control*, 22, 1–38.
- LUX, T. (1998), “The socio-economic dynamics of speculative markets: Interacting agents, chaos, and the fat tails of return distributions”, *Journal of Economic Behavior and Organization*, 33, 143–165.

- LUX, T. (2007), “Parameter estimation for stochastic models of interacting agents: An approximate ML approach via numerical solutions of transitional densities”, University of Kiel, *mimeo*.
- ORLÉAN, A. (1995), “Bayesian interactions and collective dynamics of opinion: Herd behavior and mimetic contagion”, *Journal of Economic Behavior and Organization*, 28, 257–274.
- STADLER, V. (2001), “Recovery in Germany? What sentiment indicators are telling us”, *HypoVereinsbank Policy Watch Economics*, 07/09/01; <http://www.zew.de/de/publikationen/Konjunkturerwartungen/hvb-paper.pdf>.
- TAYLOR, L. AND O’CONNELL, S. (1985), “A Minsky crisis”, *Quarterly Journal of Economics*, 100, 871–886.
- WEIDLICH, W. AND BRAUN, M. (1992), “The master equation approach to nonlinear economics”, *Journal of Evolutionary Economics*, 2, 233–265.
- WEIDLICH, W. AND HAAG, G. (1983), *Concepts and Models of a Quantitative Sociology. The Dynamics of Interacting Populations*. Berlin: Springer.
- WEISE, P. AND KRAFT, M. (1988), “Cumulative processes, disproportionalities, and Spiethoff’s theory of the business cycle”, in P. Flaschel and M. Krüger (eds), *Recent Approaches to Economic Dynamics*, pp. 3–16; Frankfurt a.M.: P. Lang.

**List of other working papers:**

**2007**

1. Timur Yusupov and Thomas Lux, The Efficient Market Hypothesis through the Eyes of an Artificial Technical Analyst: An Application of a New Chartist Methodology to High-Frequency Stock Market Data, WP07-13
2. Liu Ruipeng, Di Matteo and Thomas Lux, True and Apparent Scaling: The Proximity of the Markov- Switching Multifractal Model to Long-Range Dependence, WP07-12
3. Thomas Lux, Rational Forecasts or Social Opinion Dynamics? Identification of Interaction Effects in a Business Climate Survey, WP07-11
4. Thomas Lux, Collective Opinion Formation in a Business Climate Survey, WP07-10
5. Thomas Lux, Application of Statistical Physics in Finance and Economics, WP07-09
6. Reiner Franke, A Prototype Model of Speculative Dynamics With Position-Based Trading, WP07-08
7. Reiner Franke, Estimation of a Microfounded Herding Model On German Survey Expectations, WP07-07
8. Cees Diks and Pietro Dindo, Informational differences and learning in an asset market with boundedly rational agents, WP07-06
9. Markus Demary, Who Do Currency Transaction Taxes Harm More: Short-Term Speculators or Long-Term Investors?, WP07-05
10. Markus Demary, A Heterogenous Agents Model Usable for the Analysis of Currency Transaction Taxes, WP07-04
11. Mikhail Anufriev and Pietro Dindo, Equilibrium Return and Agents' Survival in a Multiperiod Asset Market: Analytic Support of a Simulation Model, WP07-03
12. Simone Alfarano and Michael Milakovic, Should Network Structure Matter in Agent-Based Finance?, WP07-02
13. Simone Alfarano and Reiner Franke, A Simple Asymmetric Herding Model to Distinguish Between Stock and Foreign Exchange Markets, WP07-01

**2006**

1. Roman Kozhan, Multiple Priors and No-Transaction Region, WP06-24
2. Martin Ellison, Lucio Sarno and Jouko Vilmunen, Caution and Activism? Monetary Policy Strategies in an Open Economy, WP06-23
3. Matteo Marsili and Giacomo Raffaelli, Risk bubbles and market instability, WP06-22
4. Mark Salmon and Christoph Schleicher, Pricing Multivariate Currency Options with Copulas, WP06-21
5. Thomas Lux and Taisei Kaizoji, Forecasting Volatility and Volume in the Tokyo Stock Market: Long Memory, Fractality and Regime Switching, WP06-20
6. Thomas Lux, The Markov-Switching Multifractal Model of Asset Returns: GMM Estimation and Linear Forecasting of Volatility, WP06-19
7. Peter Heemeijer, Cars Hommes, Joep Sonnemans and Jan Tuinstra, Price Stability and Volatility in Markets with Positive and Negative Expectations Feedback: An Experimental Investigation, WP06-18
8. Giacomo Raffaelli and Matteo Marsili, Dynamic instability in a phenomenological model of correlated assets, WP06-17
9. Ginestra Bianconi and Matteo Marsili, Effects of degree correlations on the loop structure of scale free networks, WP06-16
10. Pietro Dindo and Jan Tuinstra, A Behavioral Model for Participation Games with Negative Feedback, WP06-15
11. Ceek Diks and Florian Wagener, A weak bifucation theory for discrete time stochastic dynamical systems, WP06-14
12. Markus Demary, Transaction Taxes, Traders' Behavior and Exchange Rate Risks, WP06-13



13. Andrea De Martino and Matteo Marsili, Statistical mechanics of socio-economic systems with heterogeneous agents, WP06-12
14. William Brock, Cars Hommes and Florian Wagener, More hedging instruments may destabilize markets, WP06-11
15. Ginwestra Bianconi and Roberto Mulet, On the flexibility of complex systems, WP06-10
16. Ginwestra Bianconi and Matteo Marsili, Effect of degree correlations on the loop structure of scale-free networks, WP06-09
17. Ginwestra Bianconi, Tobias Galla and Matteo Marsili, Effects of Tobin Taxes in Minority Game Markets, WP06-08
18. Ginwestra Bianconi, Andrea De Martino, Felipe Ferreira and Matteo Marsili, Multi-asset minority games, WP06-07
19. Ba Chu, John Knight and Stephen Satchell, Optimal Investment and Asymmetric Risk for a Large Portfolio: A Large Deviations Approach, WP06-06
20. Ba Chu and Soosung Hwang, The Asymptotic Properties of AR(1) Process with the Occasionally Changing AR Coefficient, WP06-05
21. Ba Chu and Soosung Hwang, An Asymptotics of Stationary and Nonstationary AR(1) Processes with Multiple Structural Breaks in Mean, WP06-04
22. Ba Chu, Optimal Long Term Investment in a Jump Diffusion Setting: A Large Deviation Approach, WP06-03
23. Mikhail Anufriev and Gulio Bottazzi, Price and Wealth Dynamics in a Speculative Market with Generic Procedurally Rational Traders, WP06-02
24. Simonae Alfarano, Thomas Lux and Florian Wagner, Empirical Validation of Stochastic Models of Interacting Agents: A "Maximally Skewed" Noise Trader Model?, WP06-01

## **2005**

1. Shaun Bond and Soosung Hwang, Smoothing, Nonsynchronous Appraisal and Cross-Sectional Aggregation in Real Estate Price Indices, WP05-17
2. Mark Salmon, Gordon Gemmill and Soosung Hwang, Performance Measurement with Loss Aversion, WP05-16
3. Philippe Curty and Matteo Marsili, Phase coexistence in a forecasting game, WP05-15
4. Matthew Hurd, Mark Salmon and Christoph Schleicher, Using Copulas to Construct Bivariate Foreign Exchange Distributions with an Application to the Sterling Exchange Rate Index (Revised), WP05-14
5. Lucio Sarno, Daniel Thornton and Giorgio Valente, The Empirical Failure of the Expectations Hypothesis of the Term Structure of Bond Yields, WP05-13
6. Lucio Sarno, Ashoka Mody and Mark Taylor, A Cross-Country Financial Accelerator: Evidence from North America and Europe, WP05-12
7. Lucio Sarno, Towards a Solution to the Puzzles in Exchange Rate Economics: Where Do We Stand?, WP05-11
8. James Hodder and Jens Carsten Jackwerth, Incentive Contracts and Hedge Fund Management, WP05-10
9. James Hodder and Jens Carsten Jackwerth, Employee Stock Options: Much More Valuable Than You Thought, WP05-09
10. Gordon Gemmill, Soosung Hwang and Mark Salmon, Performance Measurement with Loss Aversion, WP05-08
11. George Constantinides, Jens Carsten Jackwerth and Stylianos Perrakis, Mispricing of S&P 500 Index Options, WP05-07
12. Elisa Luciano and Wim Schoutens, A Multivariate Jump-Driven Financial Asset Model, WP05-06
13. Cees Diks and Florian Wagener, Equivalence and bifurcations of finite order stochastic processes, WP05-05
14. Devraj Basu and Alexander Stremme, CAY Revisited: Can Optimal Scaling Resurrect the (C)CAPM?, WP05-04
15. Ginwestra Bianconi and Matteo Marsili, Emergence of large cliques in random scale-free networks, WP05-03
16. Simone Alfarano, Thomas Lux and Friedrich Wagner, Time-Variation of Higher Moments in a Financial Market with Heterogeneous Agents: An Analytical Approach, WP05-02
17. Abhay Abhayankar, Devraj Basu and Alexander Stremme, Portfolio Efficiency and Discount Factor Bounds with Conditioning Information: A Unified Approach, WP05-01

## **2004**

1. Xiaohong Chen, Yanqin Fan and Andrew Patton, Simple Tests for Models of Dependence Between Multiple Financial Time Series, with Applications to U.S. Equity Returns and Exchange Rates, WP04-19
2. Valentina Corradi and Walter Distaso, Testing for One-Factor Models versus Stochastic Volatility Models, WP04-18
3. Valentina Corradi and Walter Distaso, Estimating and Testing Sochastic Volatility Models using Realized Measures, WP04-17
4. Valentina Corradi and Norman Swanson, Predictive Density Accuracy Tests, WP04-16
5. Roel Oomen, Properties of Bias Corrected Realized Variance Under Alternative Sampling Schemes, WP04-15
6. Roel Oomen, Properties of Realized Variance for a Pure Jump Process: Calendar Time Sampling versus Business Time Sampling, WP04-14
7. Richard Clarida, Lucio Sarno, Mark Taylor and Giorgio Valente, The Role of Asymmetries and Regime Shifts in the Term Structure of Interest Rates, WP04-13
8. Lucio Sarno, Daniel Thornton and Giorgio Valente, Federal Funds Rate Prediction, WP04-12
9. Lucio Sarno and Giorgio Valente, Modeling and Forecasting Stock Returns: Exploiting the Futures Market, Regime Shifts and International Spillovers, WP04-11
10. Lucio Sarno and Giorgio Valente, Empirical Exchange Rate Models and Currency Risk: Some Evidence from Density Forecasts, WP04-10
11. Ilias Tsiakas, Periodic Stochastic Volatility and Fat Tails, WP04-09
12. Ilias Tsiakas, Is Seasonal Heteroscedasticity Real? An International Perspective, WP04-08
13. Damin Challet, Andrea De Martino, Matteo Marsili and Isaac Castillo, Minority games with finite score memory, WP04-07
14. Basel Awartani, Valentina Corradi and Walter Distaso, Testing and Modelling Market Microstructure Effects with an Application to the Dow Jones Industrial Average, WP04-06
15. Andrew Patton and Allan Timmermann, Properties of Optimal Forecasts under Asymmetric Loss and Nonlinearity, WP04-05
16. Andrew Patton, Modelling Asymmetric Exchange Rate Dependence, WP04-04
17. Alessio Sancetta, Decoupling and Convergence to Independence with Applications to Functional Limit Theorems, WP04-03
18. Alessio Sancetta, Copula Based Monte Carlo Integration in Financial Problems, WP04-02
19. Abhay Abhayankar, Lucio Sarno and Giorgio Valente, Exchange Rates and Fundamentals: Evidence on the Economic Value of Predictability, WP04-01

## **2002**

1. Paolo Zaffaroni, Gaussian inference on Certain Long-Range Dependent Volatility Models, WP02-12
2. Paolo Zaffaroni, Aggregation and Memory of Models of Changing Volatility, WP02-11
3. Jerry Coakley, Ana-Maria Fuertes and Andrew Wood, Reinterpreting the Real Exchange Rate - Yield Diffential Nexus, WP02-10
4. Gordon Gemmill and Dylan Thomas , Noise Training, Costly Arbitrage and Asset Prices: evidence from closed-end funds, WP02-09
5. Gordon Gemmill, Testing Merton's Model for Credit Spreads on Zero-Coupon Bonds, WP02-08
6. George Christodoulakis and Steve Satchell, On th Evolution of Global Style Factors in the MSCI Universe of Assets, WP02-07
7. George Christodoulakis, Sharp Style Analysis in the MSCI Sector Portfolios: A Monte Caro Integration Approach, WP02-06
8. George Christodoulakis, Generating Composite Volatility Forecasts with Random Factor Betas, WP02-05
9. Claudia Riveiro and Nick Webber, Valuing Path Dependent Options in the Variance-Gamma Model by Monte Carlo with a Gamma Bridge, WP02-04
10. Christian Pedersen and Soosung Hwang, On Empirical Risk Measurement with Asymmetric Returns Data, WP02-03
11. Roy Batchelor and Ismail Orgakcioglu, Event-related GARCH: the impact of stock dividends in Turkey, WP02-02

12. George Albanis and Roy Batchelor, Combining Heterogeneous Classifiers for Stock Selection, WP02-01

## **2001**

1. Soosung Hwang and Steve Satchell , GARCH Model with Cross-sectional Volatility; GARCHX Models, WP01-16
2. Soosung Hwang and Steve Satchell, Tracking Error: Ex-Ante versus Ex-Post Measures, WP01-15
3. Soosung Hwang and Steve Satchell, The Asset Allocation Decision in a Loss Aversion World, WP01-14
4. Soosung Hwang and Mark Salmon, An Analysis of Performance Measures Using Copulae, WP01-13
5. Soosung Hwang and Mark Salmon, A New Measure of Herding and Empirical Evidence, WP01-12
6. Richard Lewin and Steve Satchell, The Derivation of New Model of Equity Duration, WP01-11
7. Massimiliano Marcellino and Mark Salmon, Robust Decision Theory and the Lucas Critique, WP01-10
8. Jerry Coakley, Ana-Maria Fuertes and Maria-Teresa Perez, Numerical Issues in Threshold Autoregressive Modelling of Time Series, WP01-09
9. Jerry Coakley, Ana-Maria Fuertes and Ron Smith, Small Sample Properties of Panel Time-series Estimators with I(1) Errors, WP01-08
10. Jerry Coakley and Ana-Maria Fuertes, The Felsdtein-Horioka Puzzle is Not as Bad as You Think, WP01-07
11. Jerry Coakley and Ana-Maria Fuertes, Rethinking the Forward Premium Puzzle in a Non-linear Framework, WP01-06
12. George Christodoulakis, Co-Volatility and Correlation Clustering: A Multivariate Correlated ARCH Framework, WP01-05
13. Frank Critchley, Paul Marriott and Mark Salmon, On Preferred Point Geometry in Statistics, WP01-04
14. Eric Bouyé and Nicolas Gaussel and Mark Salmon, Investigating Dynamic Dependence Using Copulae, WP01-03
15. Eric Bouyé, Multivariate Extremes at Work for Portfolio Risk Measurement, WP01-02
16. Erick Bouyé, Vado Durrleman, Ashkan Nikeghbali, Gael Riboulet and Thierry Roncalli, Copulas: an Open Field for Risk Management, WP01-01

## **2000**

1. Soosung Hwang and Steve Satchell , Valuing Information Using Utility Functions, WP00-06
2. Soosung Hwang, Properties of Cross-sectional Volatility, WP00-05
3. Soosung Hwang and Steve Satchell, Calculating the Miss-specification in Beta from Using a Proxy for the Market Portfolio, WP00-04
4. Laun Middleton and Stephen Satchell, Deriving the APT when the Number of Factors is Unknown, WP00-03
5. George A. Christodoulakis and Steve Satchell, Evolving Systems of Financial Returns: Auto-Regressive Conditional Beta, WP00-02
6. Christian S. Pedersen and Stephen Satchell, Evaluating the Performance of Nearest Neighbour Algorithms when Forecasting US Industry Returns, WP00-01

## **1999**

1. Yin-Wong Cheung, Menzie Chinn and Ian Marsh, How do UK-Based Foreign Exchange Dealers Think Their Market Operates?, WP99-21
2. Soosung Hwang, John Knight and Stephen Satchell, Forecasting Volatility using LINEX Loss Functions, WP99-20
3. Soosung Hwang and Steve Satchell, Improved Testing for the Efficiency of Asset Pricing Theories in Linear Factor Models, WP99-19
4. Soosung Hwang and Stephen Satchell, The Disappearance of Style in the US Equity Market, WP99-18

5. Soosung Hwang and Stephen Satchell, Modelling Emerging Market Risk Premia Using Higher Moments, WP99-17
6. Soosung Hwang and Stephen Satchell, Market Risk and the Concept of Fundamental Volatility: Measuring Volatility Across Asset and Derivative Markets and Testing for the Impact of Derivatives Markets on Financial Markets, WP99-16
7. Soosung Hwang, The Effects of Systematic Sampling and Temporal Aggregation on Discrete Time Long Memory Processes and their Finite Sample Properties, WP99-15
8. Ronald MacDonald and Ian Marsh, Currency Spillovers and Tri-Polarity: a Simultaneous Model of the US Dollar, German Mark and Japanese Yen, WP99-14
9. Robert Hillman, Forecasting Inflation with a Non-linear Output Gap Model, WP99-13
10. Robert Hillman and Mark Salmon, From Market Micro-structure to Macro Fundamentals: is there Predictability in the Dollar-Deutsche Mark Exchange Rate?, WP99-12
11. Renzo Avesani, Giampiero Gallo and Mark Salmon, On the Evolution of Credibility and Flexible Exchange Rate Target Zones, WP99-11
12. Paul Marriott and Mark Salmon, An Introduction to Differential Geometry in Econometrics, WP99-10
13. Mark Dixon, Anthony Ledford and Paul Marriott, Finite Sample Inference for Extreme Value Distributions, WP99-09
14. Ian Marsh and David Power, A Panel-Based Investigation into the Relationship Between Stock Prices and Dividends, WP99-08
15. Ian Marsh, An Analysis of the Performance of European Foreign Exchange Forecasters, WP99-07
16. Frank Critchley, Paul Marriott and Mark Salmon, An Elementary Account of Amari's Expected Geometry, WP99-06
17. Demos Tambakis and Anne-Sophie Van Royen, Bootstrap Predictability of Daily Exchange Rates in ARMA Models, WP99-05
18. Christopher Neely and Paul Weller, Technical Analysis and Central Bank Intervention, WP99-04
19. Christopher Neely and Paul Weller, Predictability in International Asset Returns: A Re-examination, WP99-03
20. Christopher Neely and Paul Weller, Intraday Technical Trading in the Foreign Exchange Market, WP99-02
21. Anthony Hall, Soosung Hwang and Stephen Satchell, Using Bayesian Variable Selection Methods to Choose Style Factors in Global Stock Return Models, WP99-01

## **1998**

1. Soosung Hwang and Stephen Satchell, Implied Volatility Forecasting: A Comparison of Different Procedures Including Fractionally Integrated Models with Applications to UK Equity Options, WP98-05
2. Roy Batchelor and David Peel, Rationality Testing under Asymmetric Loss, WP98-04
3. Roy Batchelor, Forecasting T-Bill Yields: Accuracy versus Profitability, WP98-03
4. Adam Kurpiel and Thierry Roncalli, Option Hedging with Stochastic Volatility, WP98-02
5. Adam Kurpiel and Thierry Roncalli, Hopscotch Methods for Two State Financial Models, WP98-01